

Shepley L. Ross *Introduction to Ordinary Differential Equations*

Chapter 4. Explicit Methods of Solving Higher-Order Linear Differential Equations

4.1. Basic Theory of Linear Differential Equations

4.1.B.1. Theorem 4.1 applies to one of the following problems but not the other. Determine to which of the problems the theorem applies and state precisely the conclusion which can be drawn in this case. Explain why the theorem does not apply to the remaining problem.

(a) $y'' + 5y' + 6y = e^x$, $y(0) = 5$, $y'(0) = 7$.

(b) $y'' + 5y' + 6y = e^x$, $y(0) = 5$, $y'(1) = 7$.

4.1.B.3. Prove Theorem 4.2 for the case $m = n = 2$. That is, prove that if $f_1(x)$ and $f_2(x)$ are solutions of $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, then $c_1f_1(x) + c_2f_2(x)$ is also a solution of this equation, where c_1 and c_2 are arbitrary constants.

4.1.B.7. Consider the differential equation $y'' - 5y' + 6y = 0$.

(a) Show that e^{2x} and e^{3x} are linear independent solutions of this equation on the interval $-\infty < x < \infty$.

(b) Write the general solution of the given DE.

(c) Find the solution that satisfies the conditions $y(0) = 2$, $y'(0) = 3$. Explain why this solution is unique. Over what interval is it defined?

4.1.B.9. Consider the differential equation $x^2y'' - 2xy' + 2y = 0$.

(a) Show that x and x^2 are linearly independent solutions of this equation on the interval $0 < x < \infty$.

(b) Write the general solution of the given equation.

(c) Find the solution that satisfies the conditions $y(1) = 3$, $y'(1) = 2$. Explain why this solution is unique. Over what interval is this solution defined?

4.1.B.10. Consider the differential equation $x^2y'' + xy' - 4y = 0$.

(a) Show that x^2 and $1/x^2$ are linearly independent solutions of this equation on the interval $0 < x < \infty$.

(b) Write the general solution of the given equation.

(c) Find the solution that satisfies the conditions $y(2) = 3$, $y'(2) = -1$. Explain why this solution is unique. Over what interval is this solution defined?

4.1.D.1. Given that $y = x$ is a solution of $x^2y'' - 4xy' + 4y = 0$, find a linearly independent solution by reducing the order. Write the general solution.

4.1.D.3. Given that $y = x$ is a solution of $(x^2 - 1)y'' - 2xy' + 2y = 0$, find a linearly independent solution by reducing the order. Write the general solution.

4.1.D.9. Prove Theorem 4.8 for the case $n = 2$. That is, prove that if u is any solution of $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ and v is any solution of $a_0(x)y'' + a_1(x)y' + a_2(x)y = F(x)$, then $u + v$ is also a solution of this latter nonhomogeneous equation.