Shepley L. Ross *Introduction to Ordinary Differential Equations*

Chapter 5. Applications of Second-Order Linear Differential Equations with Constant Coefficients

5.3. Free, Damped Motion

5.3.3. An 8 lb weight is attached to the lower end of a coil spring suspended from a fixed support. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 in. The weight is then pulled down 9 in. below its equilibrium position and is released at $t = 0$. The medium offers a resistance in pounds numerically equal to $4x'$, where x' is the instantaneous velocity in feet per second. Determine the displacement of the weight as a function of time.

5.3.5. A 70 gm mass is attached to the lower end of a coil spring suspended from the ceiling. The mass comes to rest in its equilibrium position, thereby stretching the spring 5 cm. The mass is then pulled down 4 cm below its equilibrium and released at $t = 0$. A damping mechanism provides a resistance numerically equal to $280x'$ where x' is the instantaneous velocity in centimeters per second. Find the displacement of the mass as a function of time.

5.3.9. A 4 lb weight is hung upon the lower end of a coil spring hanging vertically from a fixed support. The weight comes to rest in its equilibrium position, thereby stretching the spring 8 in. The weight is then pulled down a certain distance below this equilibrium position and released at $t = 0$. The medium offers a resistance in pounds numerically equal to ax', where $a > 0$ and x' is the instantaneous velocity in feet per second. Show that the motion is oscillatory if $a < \sqrt{3}$, critically damped if $a = \sqrt{3}$, and overdamped if $a > \sqrt{3}$.