Shepley L. Ross *Introduction to Ordinary Differential Equations*

Chapter 9. The Laplace Transform

9.1. Definition, Existence, and Basic Properties of the Laplace Transform

9.1.A.1. Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ for $f(t) = t^2$.

9.1.A.2. Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ for $f(t) = \sinh t$. Recall: $\sinh x = (e^x - e^{-x})/2$ and $\cosh x = (e^x + e^{-x})/2$.

9.1.A.3. Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ for $f(t) = \begin{cases} 5 & \text{if } t \in [0,2) \\ 0 & \text{if } t \in [2,\infty). \end{cases}$

9.1.A.7. Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ for $f(t) = \begin{cases} 5 & \text{if } t \in [0,1) \\ 2-t & \text{if } t \in [1,2) \\ 0 & \text{if } t \in [2,\infty). \end{cases}$

9.1.B.1. Use Theorem 9.2 to find $\mathcal{L}\{\cos^2 at\}$. HINT: From the double angle formula for cosine we have $\cos^2\theta = \dfrac{\cos(2\theta)+1}{2}$.

9.1.B.5. If $\mathcal{L}\{t^2\} = 2/s^3$, use Theorem 9.3 to find $\mathcal{L}\{t_3\}$.

9.1.B.7. Use Theorem 9.3 and Corollary 9.1.A to find $\mathcal{L}\{f(t)\}$ if $f''(t)+3f'(t)+3f(t) = 0$, $f(0) = 1$, $f'(0) = 2$. You may assume that f satisfies the hypothesis of Theorem 9.3 and Corollary 9.1.A.

9.1.B.9. Use Theorem 9.3 and Corollary 9.1.A to find $\mathcal{L}\{f(t)\}$ if $f'''(t) = f'(t)$, $f''(0) = 2$, $f'(0) = 1$, $f(0) = 0$. You may assume that f satisfies the hypothesis of Theorem 9.3 and Corollary 9.1.A.

9.1.B.13. Use Theorem 9.5, the Translation Property, to find $\mathcal{L}\{e^{at}t^2\}$.

9.1.B.17. Use Theorem 9.6 to find $\mathcal{L}\{t^3 e^{at}\}$.