

Advanced Differential Equations

Chapter 4. Linear Operators on Hilbert Spaces Section 4.6. Positive Operators—Proofs of Theorems



Theorem 4.6.1

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Theorem 4.6.1. For any bounded linear operator A , A^*A and AA^* are positive.

Proof. For all $x \in H$,

$$(A^*Ax, x) = (Ax, Ax) = \|Ax\|^2 \geq 0$$

and

$$(AA^*x, x) = (A^*x, A^*x) = \|A^*x\|^2 \geq 0.$$

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Theorem 4.6.2

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Theorem 4.6.2. If A is invertible and positive then A^{-1} is positive.

Proof. If $y \in \mathcal{D}(A^{-1})$ then $y = Ax$ for some $x \in H$ and

$$(A^{-1}y, y) = (A^{-1}Ax, Ax) = (x, Ax) \geq 0.$$

□

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