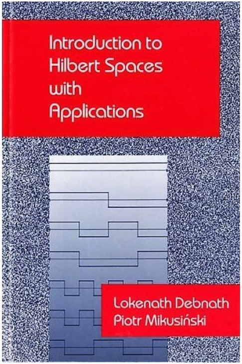
Advanced Differential Equations

Chapter 4. Linear Operators on Hilbert Spaces

Section 4.6. Positive Operators—Proofs of Theorems



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Theorem 4.6.1

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Proof. For all $x \in H$,

$$(A^*Ax, x) = (Ax, Ax) = \|Ax\|^2 \geq 0$$

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Proof. If $y \in \mathcal{D}(A^{-1})$ then $y = Ax$ for some $x \in H$ and

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$\square$