Write in complete sentences!!! Explain what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

1.1(a). Let $s$ denote the linear space consisting of all sequences over field $\mathbb{F}$. Let $c_{00}$ denote the set of all sequences that have only finitely many nonzero entries. Prove that $c_{00}$ is an infinite dimensional subspace of $s$. Find a basis of $c_{00}$. HINT: Suppose $\{b_1, b_2, \ldots, b_n\}$ is a basis and define $m_k$ to be the position in sequence $b_k$ corresponding to the last nonzero entry.

1.3(a). Given any $y \in s$, define a function $M_y : s \to s$ by $M_y(x)(n) = y(n)x(n)$. That is, $M_y$ multiplies each entry of $x$ by the corresponding entry of $y$ (it is like an entry-wise product). Prove that $M_y$ is a linear operator. HINT: Perform computations entry-wise (i.e., in terms of $n$) but draw general conclusions about elements of $s$ (i.e., sequences).

1.3(b). Describe the spaces $N(M_y)$ (the nullspace of $M_y$) and $R(M_y)$ (the range of $M_y$).

1.4(a). Define functions $S$ and $T$ from $s$ (the linear space consisting of all sequences from field $\mathbb{F}$) to itself by:
$$S(x(1), x(2), x(3), \ldots) = (0, x(1), x(2), x(3), \ldots),$$
$$T(x(1), x(2), x(3), \ldots) = (x(2), x(3), x(4), \ldots).$$
$S$ is called a right shift and $T$ is called a left shift. Prove that $S$ and $T$ are linear operators.

1.7. Suppose that for each $\lambda$ in some index set $\Lambda$ we are given a subspace $Y_\lambda$ of linear space $X$ such that for all $\lambda, \mu \in \Lambda$ we have that either $Y_\lambda \subseteq Y_\mu$ or $Y_\mu \subseteq Y_\lambda$. Prove that $\bigcup_{\lambda \in \Lambda} Y_\lambda$ is a subspace of $X$. HINT: From Linear Algebra, we know that we need only show that $\bigcup Y_\lambda$ is closed under vector addition and scalar multiplication. So show that for $y_1, y_2 \in \bigcup Y_\lambda$ and $\alpha \in \mathbb{F}$, we have $y_1 + y_2 \in \bigcup Y_\lambda$ and $\alpha y_1 \in \bigcup Y_\lambda$.

1.8. (Bonus) Let $S$ be an infinite set, and let $X$ be a linear space of functions that includes $\delta_s$ for all $s \in S$ where $\delta_s$ is the function that takes on the value 1 at $s$ and is 0 elsewhere. Prove that $X$ is infinite dimensional.