Introduction to Functional Analysis,
MATH 5740, Summer 2017

Homework 1, Chapter 1

Due Wednesday, June 7 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

1.1(a). Let s denote the linear space consisting of all sequences over field $\mathbb{F}$. Let c_{00} denote the set of all sequences that have only finitely many nonzero entries. Prove that c_{00} is an infinite dimensional subspace of s. Find a basis of c_{00}. HINT: Suppose $\{b_1, b_2, \ldots, b_n\}$ is a basis and define m_k to be the position in sequence b_k corresponding to the last nonzero entry.

1.1(b) Prove that the mapping $T : c_{00} \to \mathbb{F}$ given by $Tx = \sum_{i=1}^{\infty} x(i)$ is linear. Here, $x(i)$ is the ith entry in sequence $x \in c_{00}$. HINT: Perform computations only using finite sums.

1.3(a). Let s denote the linear space consisting of all sequences over field $\mathbb{F}$. Given any $y \in s$, define a function $M_y : s \to s$ by $M_y(x)(n) = y(n)x(n)$. That is, M_y multiplies each entry of x by the corresponding entry of y (it is like an entry-wise product). Prove that M_y is a linear operator. HINT: Perform computations entry-wise (i.e., in terms of n) but draw general conclusions about elements of s (i.e., sequences).

1.3(b). Describe the spaces $N(M_y)$ (the nullspace of M_y) and $R(M_y)$ (the range of M_y).

1.5. Prove that $\mathcal{L}(X, Z)$ is a subspace of the space of all functions from X to Z. HINT: From Linear Algebra, we know that we need only show that $\mathcal{L}(X, Z)$ is closed under vector addition and scalar multiplication. So show that for $T_1, T_2 \in \mathcal{L}(X, Z)$ and for $\alpha \in \mathbb{F}$, $T_1 + T_2$ is linear and αT_1 is linear.