Introduction to Functional Analysis,

MATH 5740, Summer 2017

Homework 2, Chapter 2, Sections 1–3

Due Tuesday, June 13 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

2.1. Let (X_1, d_1) and (X_2, d_2) be metric spaces and let $f : X_1 \to X_2$. Assume the ε/δ definition of continuity of f at point $p \in X_1$ (as in the class notes). Prove that f is continuous at $p \in X_1$ if and only if for each sequence $\{x_n\} \subset X_1$ with $x_n \to p$ we have $\lim f(x_n) = f(p)$.

2.7. Prove that in the space of bounded sequences with the sup norm, ℓ^∞, the closure of c_{00} (sequences with finitely many nonzero entries) is the subspace c_0 (sequences converging to 0).

2.9. Prove that if Y is a subspace of a normed linear space X, then Y is closed if and only if the set $A = \{y \in Y \mid \|y\| \leq 1\}$ is closed. HINT: $A \subseteq Y$ is closed if and only if $A = \overline{A}$. Use part (iii) of the Theorem about $\overline{A}$ on page 6 of the class notes for Section 2.2.

2.10. Prove that a linear operator $T : X \to Z$, where X and Z are linear spaces, is completely determined by its values on $B(x_0, r)$ for any $x_0 \in X$ and for any $r > 0$. HINT: Stretch/shrink x and translate x until it is in $B(x_0, r)$. Apply T to the result and solve for $T(x)$.