Introduction to Functional Analysis,

MATH 5740, Summer 2017

Homework 3, Chapter 2, Section 4

Due Friday, June 16 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

2.13. Let c_{00} denote the set of all sequences that have only finitely many nonzero entries. Define $T : c_{00} \to \mathbb{F}$ as $T(x) = \sum_{k=1}^{\infty} x(k)$. Let c_{00} have the sup norm. Show that T is not bounded.

2.16. Define $T : C[0,2] \to C[0,2]$ by $T(f)(t) = \int_0^t f(s)\,ds$. If the norm on $C[0,2]$ is the sup norm, what is $\|T\|$?

2.19. Recall that c_0 is the linear space of all sequences which converge to 0. Let $T \in \mathcal{B}(c_0)$ be the left-shift operator: $T(x(1), x(2), x(3), \ldots) = (x(2), x(3), x(4), \ldots)$. Prove that for all $x \in c_0$ the sequence of vectors $(T^n x)$ converges to 0 under the sup norm (on c_0), but (T^n) does not converge to 0 with respect to the norm on $\mathcal{B}(c_0)$. HINT: Use an ε argument to show $(T^n x) \to 0$. To show (T^n) does not converge to 0 in $\mathcal{B}(c_0)$, show that for all $n \in \mathbb{N}$, $\|T^n\| \geq 1$.