Introduction to Functional Analysis, MATH 5740, Summer 2017

Homework 4, Chapter 2, Sections 5 and 6

Due Tuesday, June 20 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

2.20. Consider the sup norm on c_0 and c_{00}.

    (a) Prove that in c_0 the series $\sum_{k=1}^{\infty} \delta_k/k$ is convergent but not absolutely convergent.

    (b) Give a series in c_{00} that is absolutely convergent but not convergent. HINT: Think p-series.

2.21. Consider the interval $X = [1, \infty)$ with the metric $d(x, y) = |x - y|$ and the metric $d_1(x, y) = |1/x - 1/y|$. Show the two metrics are equivalent in the sense that both have the same convergent sequences. Then show that equivalent metrics do not necessarily preserve boundedness or completeness. HINT: Suppose $(x_n) \to y$ with respect to one of the metrics. use an ε-argument, where ε is based on the value of y, to show convergence with respect to the other metric.