Introduction to Functional Analysis,

MATH 5740, Summer 2017

Homework 7, Sections 3.4, 3.5, 4.2

Due Wednesday, July 5 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

3.3. Find a sequence $y \in \ell^\infty$ such that the multiplication operator M_y on ℓ^∞ is injective (one to one) but not bounded below. HINT: Let y be any element of c_0 which is not in c_{00}.

3.6. Suppose that Y and Z are closed subspaces of a Banach space X such that $Y \cap Z = \{0\}$ and $Y + Z = X$. Define $P : X \to Y$ by $P(y + z) = y$ where $y \in Y$ and $z \in Z$. Prove that P is well defined and bounded. HINT: Prove that P is closed and use the Closed Graph Theorem.

4.2. Prove that ℓ^p for $1 \le p \le \infty$ is not an inner product space, except for $p = 2$. HINT: Use Theorem 4.8 to show that ℓ^2 is an inner product space by showing the ℓ^2 norm satisfies the Parallelogram Law. Show by example that the ℓ^p norm does not satisfy the Parallelogram Law for $p \in [1, \infty]$, $p \neq 2$.