Fundamentals of Functional Analysis MATH 5740, Summer 2021

Homework 1, Chapter 1

Due Thursday, June 10 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, notes, or hypotheses.

- **1.1.** Let s denote the linear space consisting of all sequences over field \mathbb{F} . Let c_{00} denote the set of all sequences that have only finitely many nonzero entries.
 - (a) Prove that c_{00} is an infinite dimensional subspace of s. Find a basis of c_{00} . HINT: Suppose $\{b_1, b_2, \ldots, b_n\}$ is a basis and define m_k to be the position in sequence b_k corresponding to the last nonzero entry.
 - (b) Prove that the mapping $T: c_{00} \to \mathbb{F}$ given by $Tx = \sum_{i=1}^{\infty} x(i)$ is linear. Here, x(i) is the *i*th entry in sequence $x \in c_{00}$. HINT: Perform computations only using finite sums.
- **1.4.** Define functions S and T from s (the linear space consisting of all sequences from field \mathbb{F}) to itself by:

$$S(x(1), x(2), x(3), \ldots) = (0, x(1), x(2), x(3), \ldots)$$
$$T(x(1), x(2), x(3), \ldots) = (x(2), x(3), x(4), \ldots).$$

S is called a *right shift* and T is called a *left shift*.

- (a) Prove that S and T are linear operators.
- (b) For both S and T, find the nullspace and range.