Fundamentals of Functional Analysis

MATH 5740, Summer 2021

Homework 2, Chapter 2

Due Monday, June 14 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, notes, or hypotheses.

2.3(a). Prove Proposition 2.4.

> **Proposition 2.4. (Uniqueness of Limits).** If a sequence (x_n) in a normed linear space converges to both x and y, then $x = y$.

2.6. Suppose that $\|\cdot\|$ is a seminorm on X. A seminorm satisfies all the properties of a norm EXCEPT: $\|x\| = 0 \Rightarrow x = 0$.

- **(a)** Prove that $S = \{x \in X \mid \|x\| = 0\}$ is a subspace of X. HINT: From Linear Algebra, we know that we need only show that S is closed under vector addition and scalar multiplication.
- **(b)** Prove that $\|x - y\| = 0$ implies that $\|x\| = \|y\|$.

2.9. Prove that if Y is a subspace of a normed linear space X, then Y is closed if and only if the set $A = \{y \in Y \mid \|y\| \leq 1\}$ is closed. HINT: $A \subseteq Y$ is closed if and only if $A = \overline{A}$. Use part (iii) of the Theorem 2.2.A.