Fundamentals of Functional Analysis MATH 5740, Summer 2021

Homework 3, Chapter 2

Due Thursday, June 17 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, notes, or hypotheses.

- **2.7.** Prove that in the space of bounded sequences with the sup norm, ℓ^{∞} , the closure of c_{00} (sequences with finitely many nonzero entries) is the subspace c_0 (sequences converging to 0).
- **2.11.** Prove that a sequence $(A_n) \subseteq \mathcal{B}(X, Y)$ converges to 0 if and only if for all sequences (x_n) of unit vectors we have that $(A_n x_n)$ converges to 0. HINT: Use an ε argument and the definition of A_n .
- **2.14.** Let P[0,1] denote the polynomial functions on the interval [0,1] with the sup norm. Prove that $D: P[0,1] \to P[0,1]$ where Df = df/dx, is not bounded.