

Fundamentals of Functional Analysis

MATH 5740, Summer 2021

Homework 4, Chapter 2

Due Monday, June 21 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, notes, or hypotheses.

2.15. Let $T : X \rightarrow Y$ be a linear operator. Prove $\|T\| = \sup\{\|Tx\| \mid x \in \overline{B}(1)\}$.

2.16. Define $T : C[0, 2] \rightarrow C[0, 2]$ by $T(f)(t) = \int_0^t f(s) ds$. If the norm on $C[0, 2]$ is the sup norm, what is $\|T\|$?

2.17. Suppose (S_n) is a sequence in $\mathcal{B}(X, Y)$ converging to S and (T_n) is a sequence in $\mathcal{B}(Y, Z)$ converging to T . Prove that $(T_n S_n)$ converges to TS . HINT: Use a $\varepsilon/2$ argument. You may assume a Triangle Inequality on the operator norm (see the comment on page 20 that the operator norm actually *is* a norm on $\mathcal{B}(X, Y)$).