Fundamentals of Functional Analysis

MATH 5740, Summer 2021

Homework 6, Chapter 2

Due Monday, June 28 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, notes, or hypotheses.

2.23. Prove that for X and Y are linear spaces with norms $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively, if $\|\cdot\|_1$ and $\|\cdot\|_2$ are replaced by equivalent norms $\|\cdot\|_X$ and $\|\cdot\|_Y$ respectively, then the resulting norm on $\mathcal{B}(X,Y)$ is equivalent to the original. HINT: Let $\|\cdot\|_1$ and $\|\cdot\|_X$ be equivalent norms on X, let $\|\cdot\|_2$ and $\|\cdot\|_Y$ be equivalent norms on Y, let $\|\cdot\|_3$ be the norm on $\mathcal{B}(X,Y)$ resulting from $\|\cdot\|_1$ and $\|\cdot\|_2$, and let $\|\cdot\|_B$ be the norm on $\mathcal{B}(X,Y)$ resulting from $\|\cdot\|_X$ and $\|\cdot\|_Y$. Assume $\|\cdot\|_X$ is weaker than $\|\cdot\|_1$ and $\|\cdot\|_Y$ is weaker than $\|\cdot\|_2$. Use Proposition 2.23. Then prove $\|\cdot\|_B$ is weaker than $\|\cdot\|_3$ (you will need Exercise 2.15). You get $\|\cdot\|_3$ weaker than $\|\cdot\|_B$ by interchanging the other norms.

2.30. Suppose that $T : X \to Y$ where X and Y are normed linear spaces, $T \in \mathcal{B}(X,Y)$ is bijective and bounded and T has a bounded inverse. Prove that we can replace the norm on Y by an equivalent norm so that X and Y are isometric.