Section 1.7. Completion of Normed Spaces

Note. In this section we define the completion of a normed space and the text argues that every normed space has a unique completion.

Definition. Let $(E, \|\cdot\|)$ be a normed space. The *completion* of E is the space $(\tilde{E}, \|\cdot\|_1)$ where

(a) E is a subspace of E_1,

(b) $\|x\| = \|x\|_1$ for all $x \in E$,

(c) E is dense in $\tilde{E}$ (i.e., $\text{cl}(E) = \tilde{E}$), and

(d) $\tilde{E}$ is complete.

Note. It is shown on pages 28 and 29 of the text that the completion of a normed space E exists and is unique.

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