Section 3.2. Inner Product Spaces

Note. We define an inner product space.

Definition. Let E be a complex vector space. A mapping $(\cdot,\cdot) : E \times T \to \mathbb{C}$ is an *inner product* on E if for all $x, y \in E$ and $\alpha, \beta \in \mathbb{C}$ we have:

(a) $(x, y) = \overline{(y, x)}$,

(b) $(\alpha x + \beta y, z) = \alpha(x, z) + \beta(y, z)$, and

(c) $(x, x) \geq 0$ and $(x, x) = 0$ implies $x = 0$.

A vector space with an inner product is an *inner product space* (or *pre-Hilbert space*).

Note/Definition. Property (b) is called *linearity in the first position* of the inner product. We can combine (a) and (b) to show that the inner product is *conjugate linear in the second position*:

$$(z, \alpha x + \beta y) = \overline{\alpha}(z, x) + \overline{\beta}(z, y).$$

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