Section 3.7. Orthogonal and Orthonormal Systems

Note. In this section we consider systems of mutually orthogonal unit vectors.

Definition 3.7.1. A family S of nonzero vectors in an inner product space E is an *orthogonal system* if $(x, y) = 0$ for all $x, y \in S$ with $x \neq y$. If each $x \in S$ satisfies $\|x\| = 1$, then S is an *orthonormal system.*

Theorem 3.7.1. Orthogonal systems are linearly independent.

Example 3.7.3. The *Legendre polynomials* defined by $P_0(x) = 1$,

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2 - 1)^n], \text{ for } n \in \mathbb{N}$$

form an orthogonal system in $L^2([-1, 1])$.

Note. A set of linearly independent functions can be used to generate an orthonormal set using the *Gram-Schmidt process.*

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