Chapter 4. Linear Operators on Hilbert Spaces

Note. In this chapter we cover the math necessary to do quantum mechanics. We will cover sections 4.2, 4.4–4.7, 4.8 (briefly), 4.9, and 4.10 (briefly).

Section 4.2. Examples of Operators

Note. In this section we give a standard example of an operator on the Hilbert space $\mathbb{C}^N$.

Example 4.2.2. Let A be a linear operator on $\mathbb{C}^N$, and let $\{e_1, e_2, \ldots, e_N\}$ be the standard orthonormal base of $\mathbb{C}^N$: $e_n = (0, 0, \ldots, 0, 1, 0, \ldots, 0, 0)$ where the nth entry is 1. Define for $i, j \in \{1, 2, \ldots, N\}$ as $a_{ij} = (Ae_j, e_i)$. Then for $x = \sum_{j=1}^N a_j e_j \in \mathbb{C}^N$ we have

$$Ax = A \sum_{j=1}^N a_j e_j = \sum_{j=1}^N a_j A(e_j)$$

and so

$$(Ax, e_i) = \left(\sum_{j=1}^N a_j A(e_j), e_i \right) = \sum_{j=1}^N a_j (Ae_j, e_i) = \sum_{j=1}^N a_j a_{ij}$$

where $a_{ij} = (Ae_j, e_i)$. Therefore A is represented by matrix (a_{ij}). Conversely, matrix multiplication is a linear operation.

Example. Example 4.2.3.

Note. We now skip Section 4.3, "Bilinear Functionals and Quadratic Forms," and instead cover the supplement "Linear Operators on Hilbert Spaces: Operators, Norms, Self Adjoint Operators" at `http://faculty.etsu.edu/gardnerr/talks/Butt-Head3.pdf`.

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