Section 4.4. Adjoint and Self Adjoint Operators

Note. In this section we pick up where the supplement "Linear Operators on Hilbert Spaces: Operators, Norms, Self Adjoint Operators," see `http://faculty.etsu.edu/gardnerr/talks/Butt-Head3.pdf`, left off.

Theorem 4.4.2. Let A be a bounded linear operator on a Hilbert space. Then operators $T_1 = A^*A$ and $T_2 = A + A^*$ are self adjoint.

Theorem 4.4.3. The product of two self adjoint operators is self adjoint if and only if the operators commute.

Note. If the domain of a bounded linear operator is not the whole Hilbert space, then its adjoint may not be unique. If the domain of the operator is dense in the Hilbert space, then the adjoint will be unique.

Theorem 4.4.4. Every bounded linear operator T on a Hilbert space has a representation $T = A + iB$ where A and B are self adjoint. Also, $T^* = A - iB$.

Note. As the text says, "self -adjoint operators are like real numbers in $\mathbb{C}$."

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