Section 4.6. Positive Operators

Note. In this section we define "positive operator" and state (mostly without proof) several relevant theorems.

Definition 4.6.1. A linear operator A is *positive* if it is self adjoint and $(Ax, x) \geq 0$ for all $x \in H$.

Theorem 4.6.1. For any bounded linear operator A, A^*A and AA^* are positive.

Theorem 4.6.2. If A is invertible and positive then A^{-1} is positive.

Note. If A is positive, we write $A \geq 0$. If $A - B$ are two self adjoint operators and $A - B \geq 0$ then we say $A \geq B$.

Theorem 4.6.3. The product of two commuting positive operators is a positive operator.

Definition 4.6.2. A *square root* of a positive operator A is a self adjoint operator B satisfying $B^2 = A$.

Theorem 4.6.5. Every positive operator A has a unique positive square root B. Moreover, B commutes with every operator commuting with A.

Definition 4.6.3. A self adjoint operator is *strictly positive* or *positive definite* if $(Ax, x) > 0$ for all $x \in H$ where $x \neq 0$.

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