**Section 4.6. Positive Operators**

**Note.** In this section we define “positive operator” and state (mostly without proof) several relevant theorems.

**Definition 4.6.1.** A linear operator $A$ is *positive* if it is self adjoint and $(Ax, x) \geq 0$ for all $x \in H$.

**Theorem 4.6.1.** For any bounded linear operator $A$, $A^* A$ and $AA^*$ are positive.

**Theorem 4.6.2.** If $A$ is invertible and positive then $A^{-1}$ is positive.

**Note.** If $A$ is positive, we write $A \geq 0$. If $A - B$ are two self adjoint operators and $A - B \geq 0$ then we say $A \geq B$.

**Theorem 4.6.3.** The product of two commuting positive operators is a positive operator.

**Definition 4.6.2.** A *square root* of a positive operator $A$ is a self adjoint operator $B$ satisfying $B^2 = A$. 
Theorem 4.6.5. Every positive operator $A$ has a unique positive square root $B$. Moreover, $B$ commutes with every operator commuting with $A$.

Definition 4.6.3. A self adjoint operator is strictly positive or positive definite if $(Ax, x) > 0$ for all $x \in H$ where $x \neq 0$. Revised: 4/22/2019