Applied Math I - Fall 1998

COURSE: MATH 5610-001 CALL # 31084

TIME: 11:30-12:25 MWF PLACE: Room 209 of Sam Wilson Hall

INSTRUCTOR: Dr. Robert Gardner OFFICE: Gilbreath 308L and Brown 201

OFFICE HOURS: 10:25-11:20 MWF PHONE: 439-6978 (Math Office 439-4349)

E-MAIL: gardnerr@access.etsu.edu HOMEPAGE: www.etsu.edu/math/gardner/gardner.htm

CLASS HOMEPAGE: A copy of the syllabus and additional information is available on the internet at: www.etsu.edu/math/gardner/5610/5610.htm

TEXT: *Introduction to Hilbert Spaces with Applications*, by L. Debnath and P. Mikusiński, Academic Press.

SUPPLEMENTAL TEXT: *Quantum Mechanics and Experience*, by D. Albert, Harvard University Press.

NOTE: We will spend the majority of our time on the geometry and topology of Hilbert spaces. The homework problems will include many proofs, but most are "computational proofs" and may not be quite as difficult as some of the proofs you have or will encounter in graduate school. We will spend a good bit of time (up to a month or so) on the mathematics of quantum mechanics. The level of rigor will be somewhat less for this material as compared to the earlier material.

This is the second course to be offered this (calender) year by the Math Department which addresses some of the major results in 20th century physics. The first was Differential Geometry (and Relativity), MATH 5310, which was offered Summer 1998 (see www.etsu.edu/math/gardner/5310/5310.htm for details). Those interested in *this* topic, are encouraged to participate in the Big Ugly Truly Terrible Hideous Extremely Asinine Discussion ("BUTTHEAD" for short) which is held through the Physics Department and will address gravity theory and cosmology this year (see www.etsu.edu/math/gardner/butthd.htm for details). In addition, I hope that the regular Physics Seminar will have a talk or two on quantum mechanics (see www.etsu.edu/physics/seminar.htm for details).

GRADING: Your grade will be determined by your average on homework (HW), a midterm (M), and a final (F) as follows:

$$AVERAGE = \frac{HW + M + F}{3}.$$

Grades will be assigned based on a 10 point scale with "plus" and "minus" grades being assigned as appropriate. Late homework will not be accepted. In-class presentations and class participation may also be a factor in determining grades.

Important Dates

Monday, September 7 = Labor Day holiday (no classes).
Friday, September 11 = Last day for 75% refund.
Monday, September 28 = Last day to drop without a grade of "W," last day for 25% refund.
Monday, October 26 = **Last day to drop!**
Friday, October 30 = Fall Break (no classes).
Thursday and Friday, November 26 and 27 = Thanksgiving Holiday (no classes).
Wednesday, December 9 = Last day to withdraw from the university.
Monday, December 14 = Final at 3:50-5:50 p.m.

Tentative Outline

We will cover the following chapters and topics:

Chapter 1. Normed Linear Spaces.
Complex vector spaces, function spaces, l^p spaces, bases, dimension, norms, convergence, topology, Cauchy sequences, Banach spaces, linear mappings, functionals, fixed points, Contraction Mapping Theorem.

Chapter 2. Lebesgue Integral.
Only those topics necessary for the remainder of the book will be covered.

Chapter 3. Hilbert Spaces and Orthonormal Systems.
Inner product spaces, norms, Schwarz's inequality, Hilbert spaces, Sobolev space, strong and weak convergence, orthonormal systems, Gram-Schmidt process, complete sequences, basis for Hilbert space, Fourier series, orthogonal complements, orthogonal projections, Riesz Representation Theorem, separable Hilbert spaces, Riesz-Fisher Theorem.

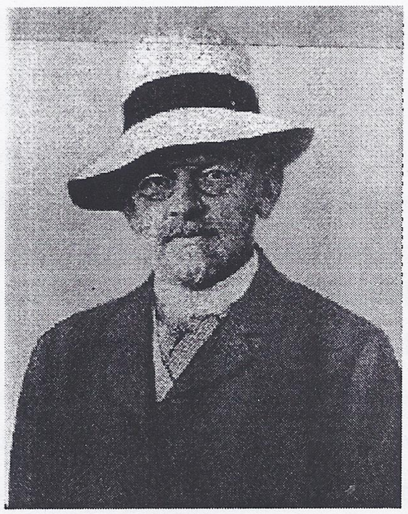
Chapter 4. Linear Operators on Hilbert Spaces.
Bounded operators, examples, bilinear functionals, adjoint and self-adjoint operators, commuting operators, inverse operators, normed operator, unitary operators, positive operators, projection operators, compact operators, eigenvalues and eigenvectors, spectrum, spectral decomposition, Fourier transform, unbounded operators.

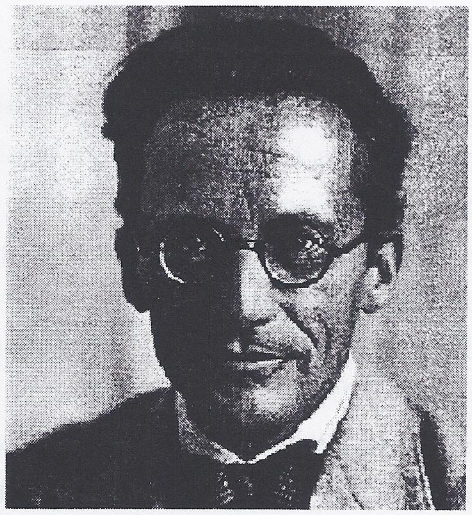
Chapter 7. Mathematical Foundations of Quantum Mechanics.

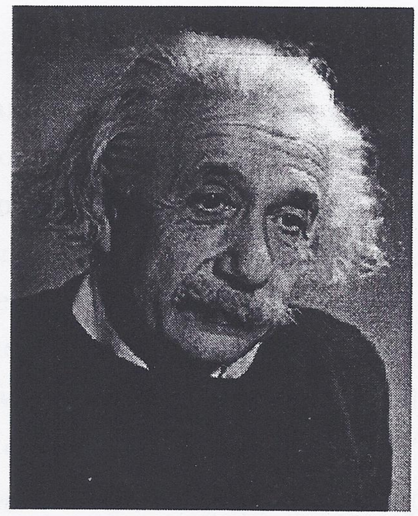
Momentum, generalized coordinates, Hamiltonian, Lagrange equation, Poisson brackets, postulates of quantum mechanics, observables, commutator, eigenstates, eigenvalues and probabilities, Heisenberg's uncertainty principle, time dependent Schrödinger equation, de Broglie waves, Schrödinger interpretation, Heisenberg interpretation, angular momentum operator, Pauli spin matrices.

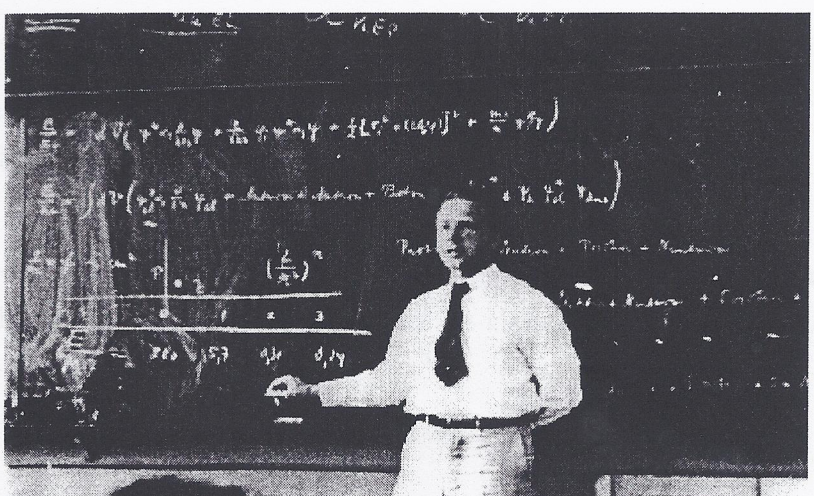
If time permits, we will also discuss topics on existence and uniqueness theorems for ordinary differential equations (see Chapter 5).

We will also discuss topics from the supplemental text, including: superposition, von Neumann's formulation, Bohm's Theory, Bell's Theorem, the Einstein-Podolsky-Rosen argument, and the Kochen-Healy-Dieks interpretation.


David Hilbert


Erwin Schroedinger


Albert Einstein


Werner Heisenberg