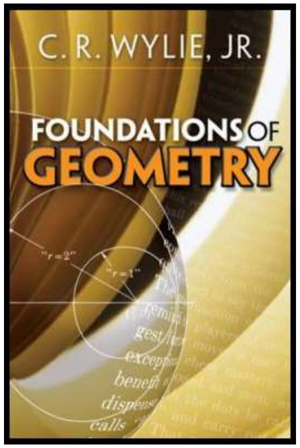
Foundations of Geometry

Chapter 2. Euclidean Geometry

2.3. The Postulate of Connection—Proofs of Theorems



Theorem 2.3.1

Theorem 2.3.1. If two lines intersect, their intersection is a point.

Proof. By Postulate 1, a line is a set of points, so a line is a set whose elements are points. By Definition 2.3.3, the intersection of two sets is the set of elements in both sets. So the intersection of two lines is the set of elements (or "points") on both lines. Since the lines intersect, the set of common points is not the empty set. Notice that the intersection cannot contain more than one point since this would imply that there is more than two points shared by the (distinct) line, in contradiction to Postulate 2. □

Theorem 2.3.4

Theorem 2.3.4. A line and a point not on the line determine a unique plane.

Proof. Let ℓ be the given line and P the given point. By Postulate 1, ℓ contains at least two points, say Q and R By hypothesis, point P is distinct from points Q and R. Points P, Q, and R are not collinear, since a line containing all three would be a second line (along with ℓ) containing both Q and R, in contradiction to Postulate 2. By Postulate 4, there is a unique plane $\mathcal{P}$ containing P, Q, and R. By Postulate 5, the plane contains line ℓ. Any plane containing line ℓ and point P, also contains points P, Q, and R, and so must be the plane $\mathcal{P}$. So $\mathcal{P}$ is the unique plane containing the line ℓ and the point P not on line ℓ. □