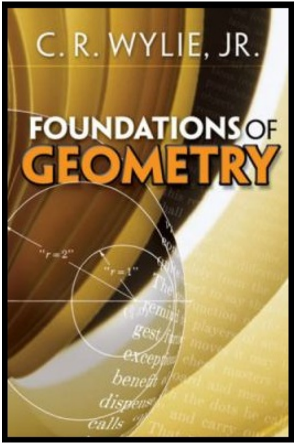
Foundations of Geometry

Chapter 2. Euclidean Geometry

2.3. The Postulate of Connection—Proofs of Theorems



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Theorem 2.3.4

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Proof. Let ℓ be the given line and P the given point. By Postulate 1, ℓ contains at least two points, say Q and R By hypothesis, point P is distinct from points Q and R. Points P, Q, and R are not collinear, since a line containing all three would be a second line (along with ℓ) containing both Q and R, in contradiction to Postulate 2.

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