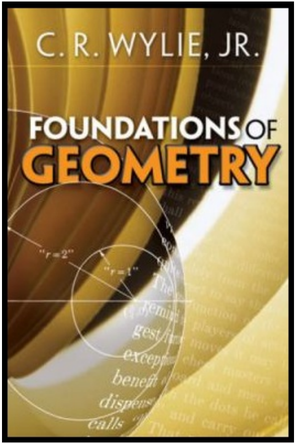
Foundations of Geometry

Chapter 2. Euclidean Geometry

2.4. The Measurements of Distance—Proofs of Theorems



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Proof. By Postulate 9 we can relate the distances between points relative to the different unit pairs as

$$m_\alpha(P,Q) = m_\alpha(B,B')m_\beta(P,Q),$$
$$m_\alpha(Q,E) = m_\alpha(B,B')m_\beta(Q,R),$$
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Substituting these relationships into $m_\alpha(P,Q) + m_\alpha(Q,R) = m_\alpha(P,R)$ gives: $m_\alpha(B,B')m_\beta(P,Q) + m_\alpha(B,B')m_\beta(Q,R) = m_\alpha(B,B')m_\beta(P,R)$.

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Substituting these relationships into $m_\alpha(P, Q) + m_\alpha(Q, R) = m_\alpha(P, R)$ gives: $m_\alpha(B, B')m_\beta(P, Q) + m_\alpha(B, B')m_\beta(Q, R) = m_\alpha(B, B')m_\beta(P, R)$. Now $m_\alpha(B, B') \neq 0$ since, by the definition of "unit pair," $B \neq B'$. So dividing through by $m_\alpha(B, B')$, the previous equation yields

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