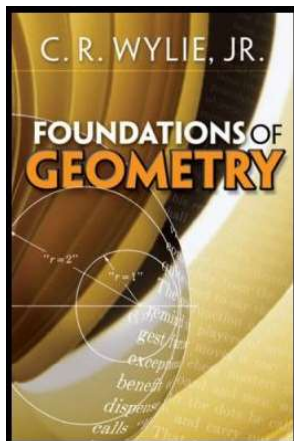


Foundations of Geometry

Chapter 2. Euclidean Geometry

2.6. Angles and Angle Measurement—Proofs of Theorems



()

Foundations of Geometry

November 7, 2021 1 / 4

Theorem 2.6.3

Theorem 2.6.3. The Angle-Construction Theorem. If H is a halfplane whose edge contains the ray \overrightarrow{VA} and if r is any number (strictly) between 0 and R , there is a unique ray \overrightarrow{VX} such that X is in H and $m_R \angle AVX = r$.

Proof. Let H be a halfplane whose edge contains the ray \overrightarrow{VA} and let r is any number (strictly) between 0 and R . By the Protractor Postulate (Postulate 15), there is a one-to-one correspondence between all numbers x for which $0 \leq x \leq R$ and the set of rays \overrightarrow{VX} which lie in the union of H and its edge, so there is a ray \overrightarrow{VX} which corresponds to r . In the Protractor Postulate, \overrightarrow{VA} corresponds to the number 0 (by part (1) of the postulate) and the ray opposite \overrightarrow{VA} corresponds the number R . Since the correspondence is one-to-one and $0 < r < R$ then the ray \overrightarrow{VX} cannot coincide with either ray \overrightarrow{VA} nor the ray opposite \overrightarrow{VA} . Therefore point X must lie in halfplane H (and not in the edge of the halfplane), as claimed.

()

Foundations of Geometry

November 7, 2021 3 / 4

Theorem 2.6.3. The Angle-Construction Theorem (continued)

Theorem 2.6.3. The Angle-Construction Theorem. If H is a halfplane whose edge contains the ray \overrightarrow{VA} and if r is any number (strictly) between 0 and R , there is a unique ray \overrightarrow{VX} such that X is in H and $m_R \angle AVX = r$.

Proof (continued). Now suppose \overrightarrow{VY} is a ray, where Y is in halfplane H , which also corresponds to the number r in the one-to-one correspondence of the Protractor Postulate. Since this correspondence is one-to-one, then it must be that ray \overrightarrow{VY} is the same as the ray \overrightarrow{VX} ; that is, ray \overrightarrow{VX} corresponding to number r is unique. \square

()

Foundations of Geometry

November 7, 2021 4 / 4