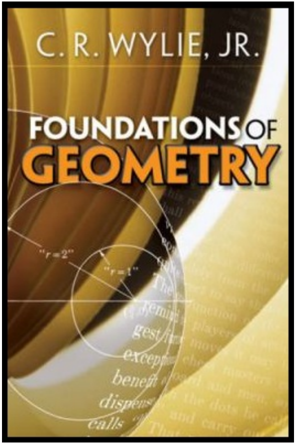
Foundations of Geometry

Chapter 2. Euclidean Geometry

2.6. Angles and Angle Measurement—Proofs of Theorems



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Theorem 2.6.3. The Angle-Construction Theorem. If H is a halfplane whose edge contains the ray $\overrightarrow{VA}$ and if r is any number (strictly) between 0 and R, there is a unique ray $\overrightarrow{VX}$ such that X is in H and $m_R\angle AVX = r$.

Proof. Let H be a halfplane whose edge contains the ray $\overrightarrow{VA}$ and let r is any number (strictly) between 0 and R. By the Protractor Postulate (Postulate 15), there is a one-to-one correspondence between all numbers x for which $0 \leq x \leq R$ and the set of rays $\overrightarrow{VX}$ which lie in the union of H and its edge, so there is a ray $\overrightarrow{VX}$ which corresponds to r.

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Theorem 2.6.3. The Angle-Construction Theorem (continued)

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Proof (continued). Now suppose $\overrightarrow{VY}$ is a ray, where Y is in halfplane H, which also corresponds to the number r in the one-to-one correspondence of the Protractor Postulate. Since this correspondence *is* one-to-one, then it must be that ray $\overrightarrow{VY}$ is the same as the ray $\overrightarrow{VX}$; that is, ray $\overrightarrow{VX}$ corresponding to number r is unique. □