Chapter 1. Elements of Geometry

Note. This chapter introduces sets and uses them to deal with lines (in particular, the number line), planes, and angles. We also consider the length of a line segment and the measure of an angle.

1.1. The Language of Sets

Note. We treat the well-known geometric figures of lines, planes, angle, and circles as "sets" of points. In this section we discuss sets rather informally. A more detailed approach to sets can be found in my online notes on Naive Set Theory, and a much more detailed and axiomatic approach can be found in my online notes for Introduction to Set Theory.

Definition 1.1.A. A *set* is a collection of objects. The objects in the set are called *members* or *elements*. The elements of a set are said to *belong to* or to be *contained in* the set.

Note. Definition 1.1.A has used the (mathematically) undefined terms "collection" and "object." Since we can only define things in terms of other things, then we must leave some terms undefined. In a modern version of university-level geometry, the terms "set" and "point" are left undefined. We will explore this some more in our Section 1.5. Basic Undefined Terms. For more details on this idea, see my online notes for Introduction to Modern Geometry (MATH 4157/5157) on Section 1.3. Axiomatic Systems.

Examples. One way to present a set is to list its elements. We use the notation of enclosing the elements with set brackets { and }. For example, consider $A = \{2, 4, 6\}$. This is read "set A with elements 2, 4, and 6." This technique of describing set A is called *specifying by roster*. Another way to present a set if by *description* or *rule*. For example, we could have

$$B = \{\text{the real numbers strictly between 0 and 1}\} = \{x \in \mathbb{R} \mid 0 < x < 1\}.$$

The little vertical line | in the right-hand description of set B is read "such that" and introduces the rule (sometimes this is replaced with a colon, as is done in the text book).

Note. As suggested above, we denote the fact that x is an element of set A as $x \in A$. If x is not an element of A, then we write $x \notin A$.

Definition 1.1.B. If the elements of a set can be counted with the counting process coming to an end, then the set is a *finite set*. Otherwise, it is an *infinite set*. A set containing no elements is the *empty set*, denoted { }, $\emptyset$, or $\varnothing$. Two sets A and B are *equal* if they contain exactly the same elements, denoted $A = B$.

Note. We largely use the following notations. We denote sets with capital italicized letters, such as A and B. We denote the real numbers with the common "black board font" $\mathbb{R}$ (as opposed to the calligraphy-type font used in the text book), and we prefer the symbol $\varnothing$ over $\emptyset$ to represent the empty set. The book often refers to "members" of sets, but we more commonly use the term "elements" of sets in these notes.

Note. To help with meaning and reading of the symbols we have introduced, we present several examples:

NOTATION	**HOW TO READ**
$x \in A$	x is an element of set A.
$x \notin A$	x is not an element of set A.
$\{3, 4, 5, \ldots, 25\}$	The set whose elements are 3, 4, 5, and so on through 25.
$\{1, 2, 3, \ldots\}$	The set whose elements are 1, 2, 3, and so on indefinitely.
$\{x \mid x \in A\} = \{x : x \in A\}$	The set of all x such that x is an element of set A
$\{x \mid 2x + 8 = 12\}$ $= \{x : 2x + 8 = 12\}$	The set of all x such that the sum of twice x and 8 is equal to 12.
$x \in \mathbb{R}$	x belongs to the set of real numbers.

Exercise 1.1.6. Use the roster method to specify the set {the positive integers}. Is the set finite or infinite?

Solution. The set of integers can be presented using the roster method as

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

To get the positive integers, we exclude the negative integers, $\ldots, -3, -2, -1$, and integer 0 (recall that 0 is neither positive nor negative). So the given set is $\boxed{\{1, 2, 3, \ldots\}}$, read "The set whose elements are 1, 2, 3, and so on indefinitely." This is an $\boxed{\text{infinite set}}$ since the process of counting the elements does not come to an end. $\square$

Exercise 1.1.22. Use the description method (or the "rule method") to specify the set {Saturday, Sunday}.

Solution. There are many possible solutions to this. Two are the following:

$$\boxed{\{\text{ Saturday, Sunday}\} = \{\text{ the days of the week which start with the letter S}\}}$$

and

$$\boxed{\{\text{ Saturday, Sunday}\} = \{\text{ the days of the week which in the weekend}\}.}$$

We include this example to show that sets can have as elements something other than numbers. □

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