1.2. Relationships between Sets

Note. We define the intersection and union of two sets, and illustrate these ideas with examples and illustrations.

Definition 1.2.A. For sets A and B, if every element of A is also an element of B then A is a *subset* of B, denoted $A \subset B$ (read "A is a subset of B"). If A is not a subset of B, we write $A \not\subset B$.

Example 1.2.1. List all subsets of $\{1, 2, 3\}$.

Solution. First, the empty set is a subset of all sets. Also, any set itself is a subset of a given set. So here there are subsets containing no elements, one element, two elements, and three elements. So the subsets are:

$$\boxed{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}.}$$

Definition 1.2.B. The *intersection* of sets A and B is the set of elements belong to both A and B. This set is denoted $A \cap B$. If A and B do not intersect (that is, if the intersection is the empty set), then these sets are *disjoint*.

Example 1.2.2. Let $A = \{2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{4, 5\}$. Find $A \cap B$, $A \cap C$, and $B \cap C$.

Solution. We have, by the definition of intersection, that

$$\boxed{A \cap B = \{2, 3\},\ A \cap C = \varnothing, \text{ and } B \cap C = \{4\}.}$$

Definition 1.2.C. The *union* of sets A and B is the set of elements belong at least one of sets A and B. This set is denoted $A \cup B$.

Example 1.2.3. Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $C = \varnothing$. Find $A \cup B$, $B \cup C$, and $(A \cup B) \cup C$.

Solution. We have, by the definition of union, that

$$\boxed{A \cup B = \{1, 2, 3, 4\},\ B \cup C = \{2, 3, 4\}, \text{ and } (A \cup B) \cup C = \{1, 2, 3, 4\}.}$$

Note. It is common to represent a set as a region of in the plane. Then intersections and unions can be represented by shading. In the following figures (from page 7 of the book), sets R and S are represented as regions, and $R \cap S$ (left) and $R \cup S$ (right) are represented by shading. We explore this idea more in the next section.