1.2. Relationships between Sets

Note. We define the intersection and union of two sets, and illustrate these ideas with examples and illustrations.

**Definition 1.2.A.** For sets $A$ and $B$, if every element of $A$ is also an element of $B$ then $A$ is a *subset* of $B$, denoted $A \subset B$ (read “$A$ is a subset of $B$”). If $A$ is not a subset of $B$, we write $A \not\subset B$.

**Example 1.2.1.** List all subsets of $\{1, 2, 3\}$.

**Solution.** First, the empty set is a subset of all sets. Also, any set itself is a subset of a given set. So here there are subsets containing no elements, one element, two elements, and three elements. So the subsets are:

$\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.

**Definition 1.2.B.** The *intersection* of sets $A$ and $B$ is the set of elements belong to both $A$ and $B$. This set is denoted $A \cap B$. If $A$ and $B$ do not intersect (that is, if the intersection is the empty set), then these sets are *disjoint*.

**Example 1.2.2.** Let $A = \{2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{4, 5\}$. Find $A \cap B$, $A \cap C$, and $B \cap C$. 
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Solution. We have, by the definition of intersection, that

\[ A \cap B = \{2, 3\}, \quad A \cap C = \emptyset, \quad \text{and} \quad B \cap C = \{4\}. \]

Definition 1.2.C. The union of sets \( A \) and \( B \) is the set of elements belong at least one of sets \( A \) and \( B \). This set is denoted \( A \cup B \).

Example 1.2.3. Let \( A = \{1, 2, 3\} \), \( B = \{2, 3, 4\} \), and \( C = \emptyset \). Find \( A \cup B \), \( B \cup C \), and \( (A \cup B) \cup C \).

Solution. We have, by the definition of union, that

\[ A \cup B = \{1, 2, 3, 4\}, \quad B \cup C = \{2, 3, 4\}, \quad \text{and} \quad (A \cup B) \cup C = \{1, 2, 3, 4\}. \]

Note. It is common to represent a set as a region of in the plane. Then intersections and unions can be represented by shading. In the following figures (from page 7 of the book), sets \( R \) and \( S \) are represented as regions, and \( R \cap S \) (left) and \( R \cup S \) (right) are represented by shading. We explore this idea more in the next section.

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