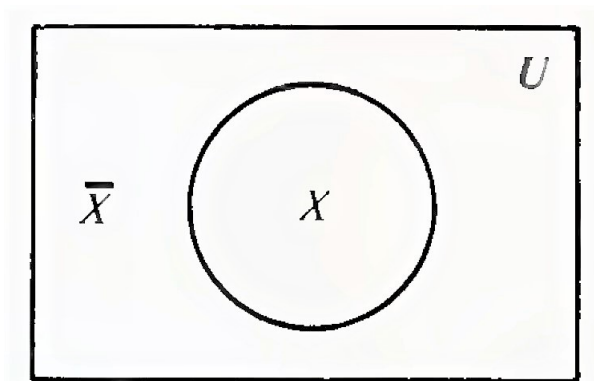


## 1.3. Venn Diagrams

**Note.** In the previous section, we represented the intersection  $R \cap S$  and union  $R \cup S$  or two sets  $R$  and  $S$  with a picture where the sets are drawn as regions in the plane. These are examples of *Venn diagrams*. They are named for John Venn (August 4, 1834–April 4, 1923), who used them in his 1881 *Symbolic Logic*. Similar ideas of representing collections of objects graphically had been proposed by others predating Venn’s work, but his book popularized the idea.

**Definition 1.3.A.** In these notes, when dealing with sets, we assume some *universal set*  $U$  that contains all objects under consideration. For a given set  $X$ , all elements in  $U$  that are not in  $X$  form the *complement* of  $X$ , denoted  $\bar{X}$ .

**Example 1.3.A.** A Venn diagram for sets  $U$ ,  $X$ , and  $\bar{X}$  is:



**Example 1.3.1.** Let  $U$  be the set of all girls,  $X$  the set of all girls who are exactly 16 years old,  $Y$  the set of girls with long hair, and  $Z$  the set of all girls who are

exactly 17 years old. (This is example from the text book, in which the terms are “quaint”; we assume the sets can be unambiguously determined in spite of the informal terminology.) Describe in words the following sets and draw a Venn diagram for each:  $\overline{X}$ ,  $X \cap Y$ ,  $X \cup Y$ ,  $X \cap Z$ ,  $X \cup Z$ , and  $\overline{X \cap Y}$ .

**Solution.** Since  $U$  is the set of all girls and  $X$  is the set of all girls who are exactly 16 years old, then

$\overline{X}$  is the set of all girls who are NOT exactly 16 years old.

Since  $X$  the set of all girls who are exactly 16 years old AND  $Y$  the set of girls with long hair, then

$X \cap Y$  is the set of all girls who are exactly 16 years old AND have long hair.

Since  $X$  the set of all girls who are exactly 16 years old and  $Y$  the set of girls with long hair, then

$X \cup Y$  is the set of all girls who are exactly 16 years old OR who have long hair.

Since  $X$  the set of all girls who are exactly 16 years old and  $Z$  is the set of all girls who are exactly 17 years old, then  $X \cap Z$  is the set of all girls who are exactly 16 years old AND are exactly 17 years old. However, no one can have two different ages, so  $X \cap Z = \emptyset$ .

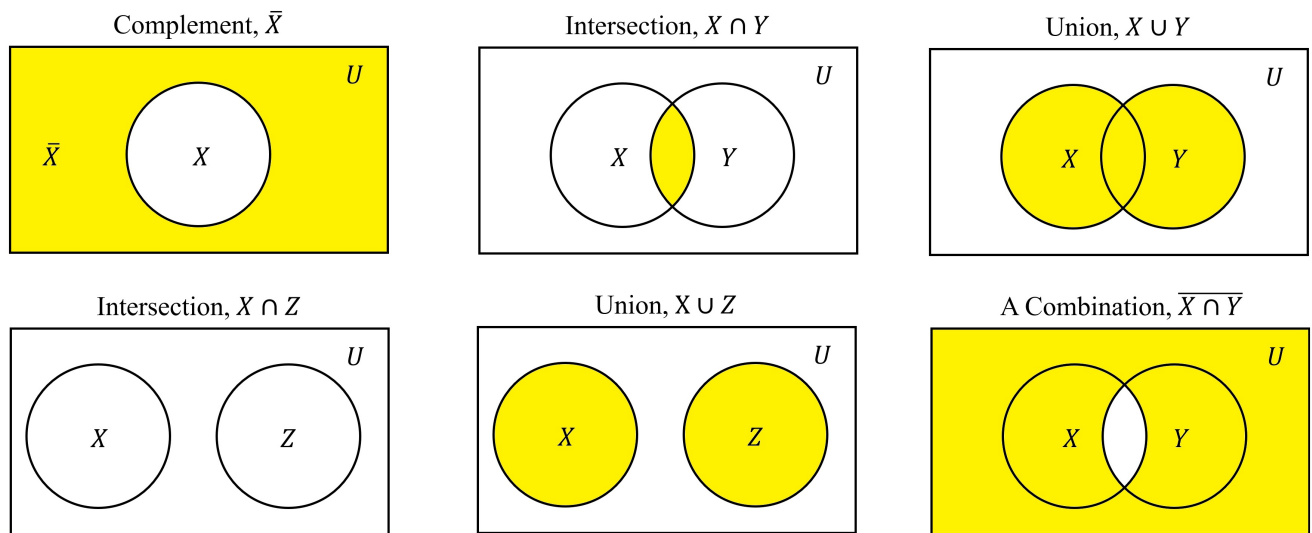
Since  $X$  the set of all girls who are exactly 16 years old and  $Z$  is the set of all girls who are exactly 17 years old, then

$X \cup Z$  is the set of all girls who are exactly 16 years old OR are exactly 17 years old.

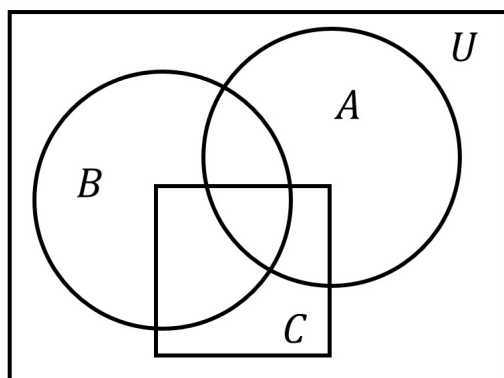
Now  $X \cap Y$  is the set of all girls who are exactly 16 years old AND have long hair, as shown above. So  $\overline{X \cap Y}$  is the set of all girls not in  $X \cap Y$ ; that is,

$\overline{X \cap Y}$  is the set of all girls who are  
EITHER not exactly 16 years old OR do not have long hair.

The Venn diagrams for these sets are:

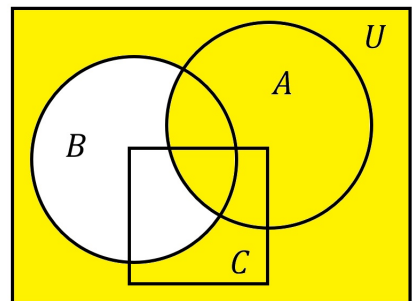
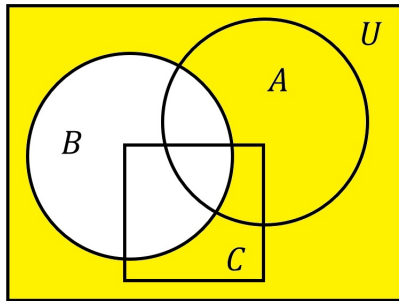
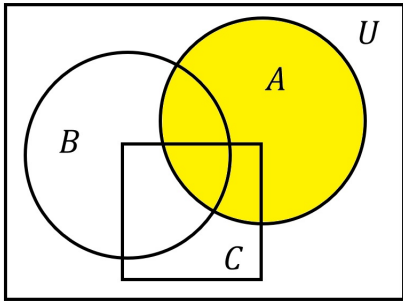


**Written Exercise 1.3.16.** Shade the regions representing  $\{x \mid x \in A \text{ or } x \in \bar{B}\}$ .



**Solution.** We break this into three parts. On the left we have shaded all  $x \in A$ .

In the middle we have shaded all  $x \in \overline{B}$ . On the right we have shaded the union (“or”) of these two sets.



*Revised: 1/5/2023*