Chapter 1. Thales and Pythagoras

Note. In this chapter, we mostly encounter results concerning lengths, angles, and areas. Throughout the notes for this book, we rely on the MacTutor History of Mathematics Archive. This is a reliable source maintained by the School of Mathematics and Statistics at the University of St. Andrews.

Section 1.1. Thales' Theorem.

Note. Thales of Miletus (624 BCE–547 BCE) lived in what is now modern-day Turkey, and traveled to Babylon and Egypt. He is known as the first Greek philosopher, scientist, and mathematician, though he worked as an engineer. None of his writing survives, but Proclus (writing in the 5th century CE) claims that Thales went to Egypt and used geometry to measure the heights of the pyramids by measuring shadows. Other references to this exist which are somewhat older; For example, Hieronymus (a student of Aristotle [384 BCE–322 BCE] also reports this). He is one of the "Seven Sages of Greece," so-named by the classical Greek philosophers (in particular, Plato). Herodotus (484 BCE–425 BCE) says that Thales predicted a solar eclipse of 585 BCE (though some skepticism exists of this claim). Several sources (primarily Proclus) imply that Thales is responsible for the following geometric theorems:

(i) A circle is bisected by any diameter.

(ii) The base angles of an isosceles triangle are equal.

(iii) The angles between two intersection straight lines are equal.

(iv) Two triangles are congruent if they have two angles and one side equal.

(v) An angle in a semicircle is a right angle.

Thales of Miletus (624 BCE–547 BCE)

This biographical information and the image of the Thales stamp is from the MacTutor History of Mathematics Archive biography of Thales (accessed 7/23/2021).

Note. The idea of measuring heights using angles and distances is related to the concept of similar triangles. Ostermann and Wanner give suggestive argument based on stretching the lengths of the sides of a triangle and then subdividing the lengths of sides to argue that the following result holds when the sides of a triangle are "extended" by a positive rational amount. The claim holds in general, but we postpone a proof until Section 2.3. Books V and VI. Real Numbers and Thales' Theorem, when we have a set of axioms and theorems on which we can base proofs.

Theorem 1.1. Thale's Intercept Theorem.

Consider an arbitrary triangle ABC (see Figure 1.3, left) and let AC be extended to C' and AB be extended to B', so that $B'C'$ is parallel to BC. Then the lengths of the sides satisfy the relations

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c} \text{ and } \frac{a'}{c'} = \frac{a}{c},\ \ \frac{c'}{b'} = \frac{c}{b},\ \ \frac{b'}{a'} = \frac{b}{a}.$$

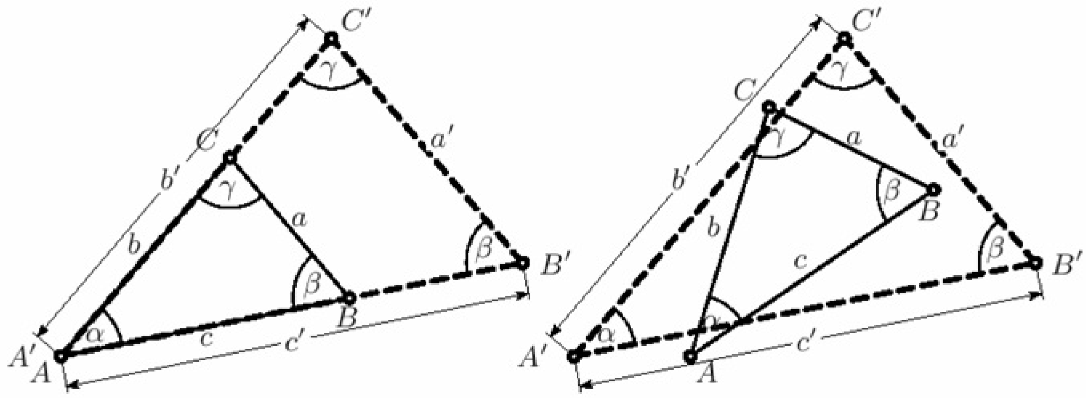

Fig. 1.3. Thales' intercept theorem

Note. The proportions are also preserved under displacements and rotations (that is, the proportions are preserved under rigid transformations); see Figure 1.3, right. So we more precisely have that: "If corresponding angles ot two triangles are equal, then corresponding sides are proportional." Such triangles are said to be *similar*. Similar triangles are unique to Euclidean geometry (that is, geometry in which the parallel postulate holds). We will see later that in hyperbolic and elliptical geometry, two triangles are similar if and only if they are congruent (that is, their corresponding angles and corresponding edges are equal).