Section 1.2. Similar Figures.

Note. We very briefly consider generalizing Thales' idea of similar triangles and constructing (in the spirit of compass and straight-edge style) rational numbers.

Note. In the works of Christopher Clavius (1538–1612) and François Viète (1540–1603), a generalization of Thales' Theorem on similar triangles appears. Two figures are *similar with similarity center* O when corresponding points A_i, B_i C_i lie on lines through O then the corresponding lines A_iA_j, B_iB_j, C_iC_j are parallel. Thales' Theorem then implies that pairs of triangles with a vertex at O implies that corresponding lengths of similar figures are proportional. See Figure 1.4.

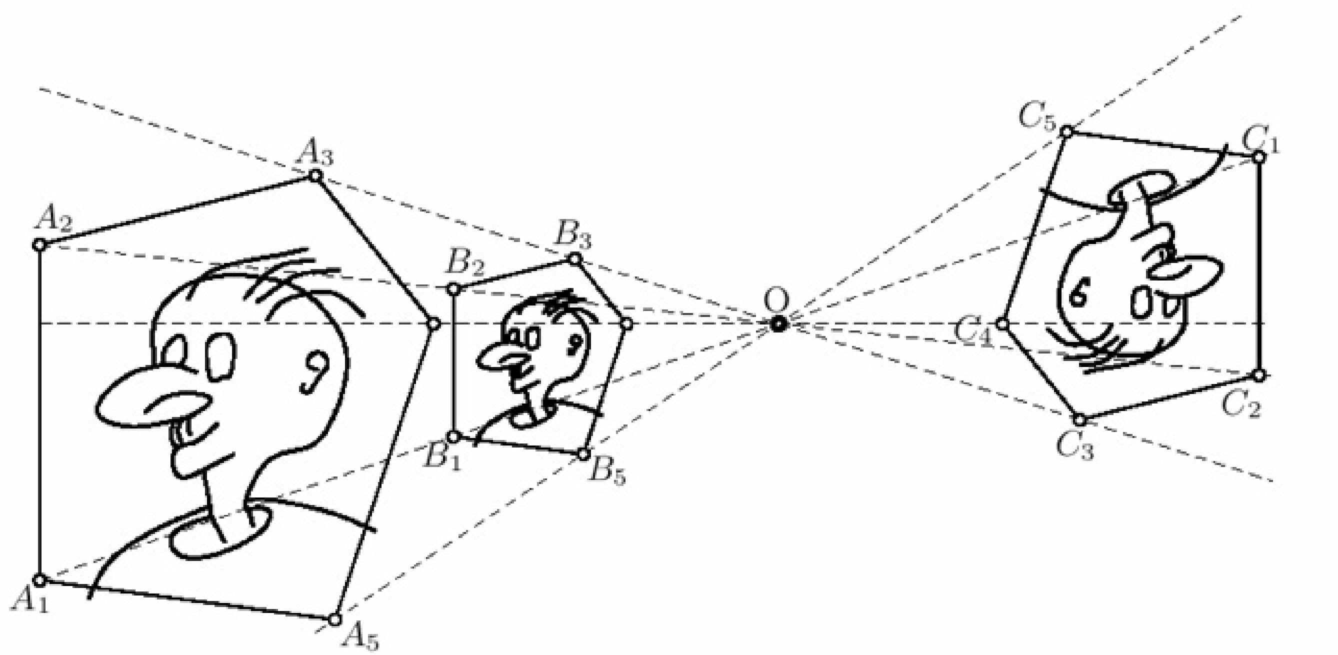

Fig. 1.4. Similar figures: illustration inspired by Clavius and Viète, improved by modern computer technology

Note. In the spirit of compass and straight-edge construction, we start with two given points which are used to define the unit length. We can reproduce this length along a line to produce a line segment of length n for any natural number n (or equivalently, we can find two points which are a distance n apart). We can then subdivide a known distance as illustrated in Figure 1.6. We take any length (length a in the figure) and repeat it several times (five times in the figure). We then use parallel line segments to subdivide the given distance into the chosen number of parts. In Figure 1.6, the unit length is divided into 5 parts resulting in distances 1/5, 2/5, 3/5, and 4/5. Similarly, for any positive rational number p/q, we can construct distance $1/q$ by partitioning the unit length into q parts, and then taking p copies of this length laid out along a line.

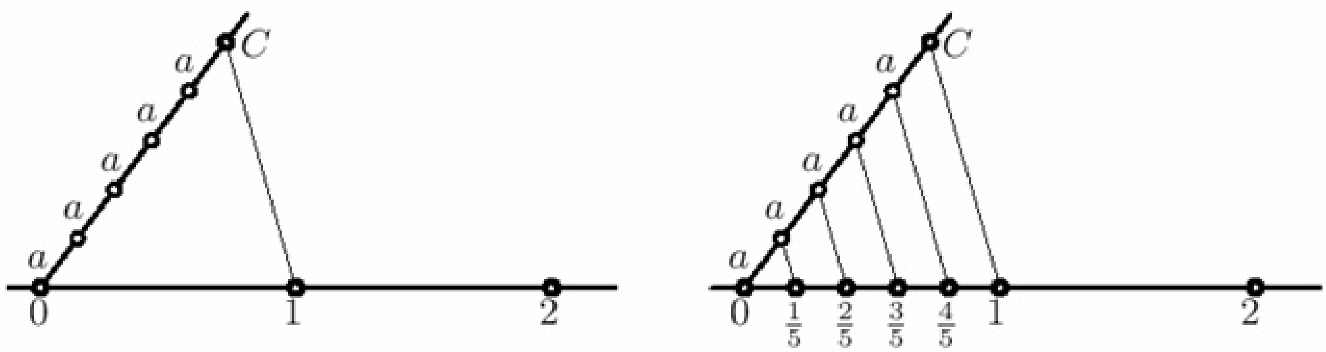

Fig. 1.6. Constructing rational lengths