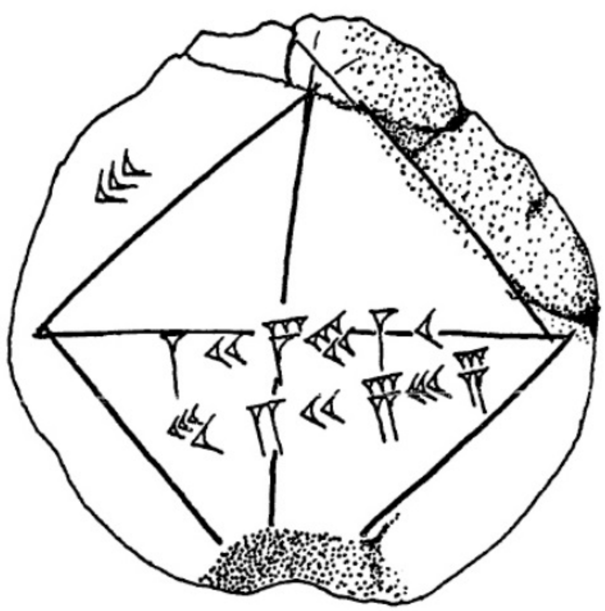
Section 1.6. A Remarkable Babylonian Document.

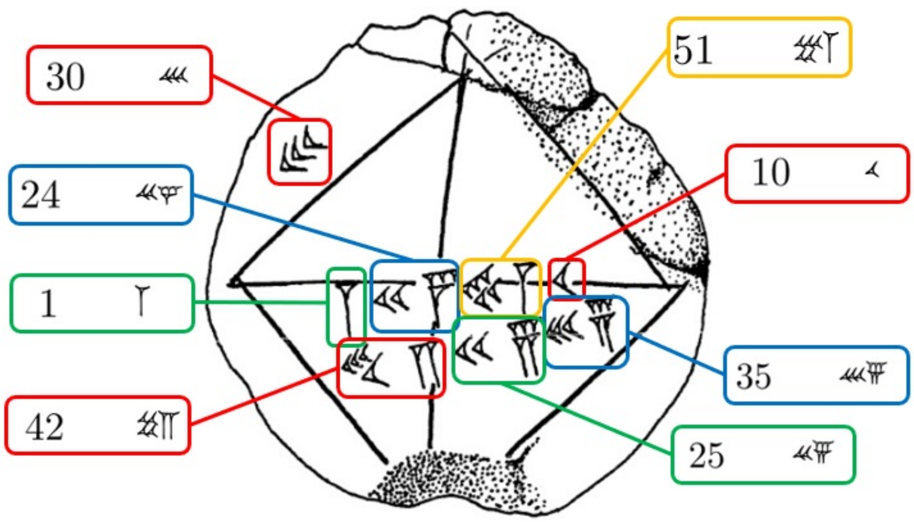
Note. In this section, we consider a numerical approximation of the length of the diagonal of a square and, hence, an approximation of $\sqrt{2}$. The computation was performed somewhere between 1900 to 1600 BCE in southern Mesopotamia and is preserved on a clay tablet about 3 inches in diameter (the tablet is denoted YBC 7289). It is now part of the Yale Babylonian Collection. It was donated to the collection by J. P. Morgan (this information is from the Wikipedia webpage on YBC 7289.

Note. The tablet appears to be a practice school exercise undertaken by a student. Details on attempts (with successes and failures) to use images of the tablet in the contemporary classroom environment are described in Janet L. Beery and Frank J. Swetz, "The Best Known Old Babylonian Tablet?," *Convergence* (July 2012).



From the MAA page on "The Best Known Old Babylonian Tablet?"

From Wikipedia page on YBC 7289



A modification of one of the above figures giving the "translation" of the Babylonian numerals into Arabic numerals.

Note. There are three numbers on the tablet. The square is meant to have sides of length 30, as labeled in the upper left. Written on and somewhat under the horizontal diagonal of the square is 1 24 51 10. Converting to Babylonian base 60 (with an implied decimal point after the 1) we have the number :

$$1 \times (60^0) + 24 \times \left(\frac{1}{60}\right) + 51 \times \left(\frac{1}{60^2}\right) + 10 \times \left(\frac{1}{60^3}\right) \approx 1.414212963.$$

Now $\sqrt{2} \approx 1.414213562$, so the horizontal diagonal seems to be labeled with a very good approximation of $\sqrt{2}$ (accurate to 6 decimal places). Under the diagonal is the result of a computation:

$$42 \times (60^0) + 25 \times \left(\frac{1}{60}\right) + 35 \times \left(\frac{1}{60^2}\right) \approx 42.426389.$$

We have $30\sqrt{2} \approx 42.426407$, so it is this number that represents the length of the diagonal, given the length of a side of the square is 30 (accurate to 4 decimal places).

Note. Ostermann and Wanner (see page 14) argue that this is evidence that the Babylonians knew the Pythagorean Theorem (over 1,000 years before the Pythagoreans), at least in the special case of an isosceles right triangle. They were also aware of the rules of proportion for similar triangles, based on the fact that $\sqrt{2}$ is the length of an isosceles right triangle with leg of length 1, and the tablet involves the computation of the length of the hypotenuse of an isosceles right triangle with leg of length 30. Plus, of course, there is also the impressive level of accuracy in the approximation of $\sqrt{2}$!

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