Chapter 2. The Elements of Euclid

Note. In this chapter, we survey the results given in the 13 books of Euclid's *Elements of Geometry*. Written about 2,300 years ago, it has been published in thousands of different editions. It is number two in terms of editions of a single work published, being only second to the Bible (according to Ian Stewart's *Why Beauty is Truth: A History of Symmetry*, Basic Books [2007]; see page 19). Its format is mimicked today in almost every upper-level math book. More than any other single work, Euclid's Elements has influenced what mathematics is and how it is done.

Note. Very little is known about Euclid the person. It is estimated that he lived from circa 325 BCE to circa 265 BCE. What is known is largely based on commentaries by Proclus, who lived around 450 CE. It has been theorized that instead of working alone, Euclid was the leader of a team that contributed to the the Elements (and other works attributed to Euclid. It has even been suggested that Euclid was not a historical figure, but instead his "writings" were created by a team of mathematicians who credited their work to the single character "Euclid" (this is not widely viewed as likely, though there is historical precedent in the books of Bourbaki which were written in the 20th century by a team and presented as works by "Bourbaki" in honor of a 19th century French general). This historical information is based on the MacTutor History of Mathematics Archive biography of Euclid of Alexandria.

Statue of Euclid at the Oxford University Museum of Natural History

Image from the Wikipedia page on Euclid (accessed 11/28/2021)

Note. The *Elements* consist of 13 books. Broadly, Books I–VI deal with plane geometry, Books VII–X with arithmetic, and Books XI–XIII cover solid geometry (including the construction of the five famous platonic solids). The size of the books varies between about 2.5% of the whole for the smallest, Book II, and 25% for Book X. Each of the others is roughly 5-8% of the total.

Note. Euclid is known to have written several other works, including:

- The *Data*. This concerns elementary geometry and may be thought of as elementary exercises in analysis.
- The book *On Divisions (of Figures)*. This work is lost in Greek but has been

discovered in the Arabic.

- The *Porisms*. This contains 38 lemmas and 171 theorems. A "porism" is something between a theorem and a problem: it deals with something already existing, as a theorem does, but has to find it (eg. the center of a circle).
- The *Conics*. This is lost, but is said to have consisted of four books and was used by Apollonius.
- The *Phaenomena*. This is an astronomical work and is still extant.
- The *Optics*. Copies still exist.
- *Elements of Music* is a work credited to Euclid, but no longer exists.

This information is from Sir Thomas Heath's *The Thirteen Books of Euclid's Elements*, 2nd Edition Revised with Additions (Cambridge University Press: 1926); see pages 8 and 17.

Note. Proclus lived from 410 to 485 CE. Therefore, his work dates from around 700 years after that of Euclid. Proclus wrote an extensive commentary on Book I of Euclid's *Elements*. His commentary is one of the two main sources of information as to the history of Greek geometry which we possess, the other being the Collection of Pappus (see Heath's *The Thirteen Books of Euclid's Elements*, page 29). In fact, Proclus' commentary on the Elements is still in print today.

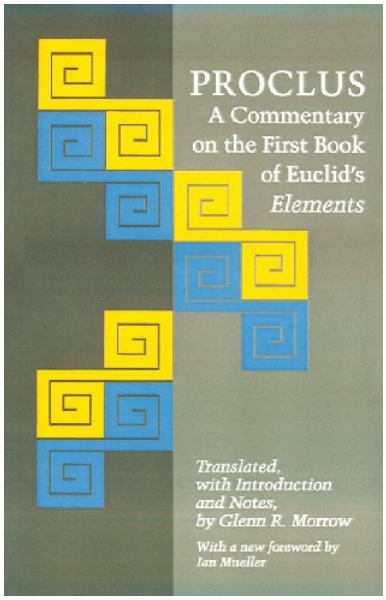


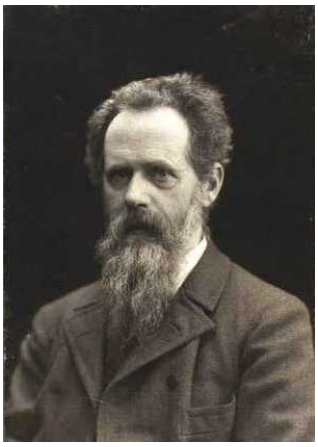
Image from the Amazon.com (accessed 11/28/2021)

Note. As Europe fell into the dark ages, the work of the classical period was preserved in the Arabic world. By the middle of the eighth century, all the important work in mathematics was being done by Islamic scholars. This would remain the case for the next several hundred years. Scholarship exploded in the Middle East, fueled in part by the availability of numerous ancient texts from Byzantium and elsewhere. In the ninth century and afterward, many of the classical works of the ancient world were translated from Greek into Arabic (see page 62 of Jason Socrates Bardi's *The Fifth Postulate: How Unraveling a Two-Thousand-Year-Old Mystery Unraveled the Universe*, John Wiley & Sons [2009]). Heath's *The Thirteen Books of Euclid's Elements* introduction lists 31 Arabic translators of The Elements. The first printed version of The Elements appeared in 1482 in Venice. The text was based upon a translation from Arabic to Latin presumably made by Abelard of Bath in the 12th century, edited and annotated by Giovanni Compano.

It included over 400 figures. The first English translation of the Elements was by Sir Henry Billinsgley and appeared in London in 1570. It includes Books I-XV, along with a Book XVI by Candalla. The notes include all the most important from the Greek commentaries of Proclus and the others, down to commentaries of the time (see Heath's *The Thirteen Books of Euclid's Elements*, pages 109 and 110). Additional details on the history of the *Elements* is in my online presentation Euclid's Elements—A 2,500 Year History. This presentation also includes information on copies of the *Elements* at the ETSU Sherrod Library, hard copies available for sale, and links to a number of online versions.

Note. The most popular and widely available version of The Elements today is the so-called "Heiberg/Heath" translation. Johan Ludvig Heiberg lived from 1854 to 1928. He was a professor of Classical Philology at the University of Copenhagen. His publications (which number over 200) include translations of Archimedes, Apollonius of Perga, and, most important to us, Euclid. He also studied the famous Archimedes Palimpsest in 1906. He studied several extant editions of The Elements when writing his *Euclidis Elementa* which was published between 1883 and 1888. Sir Thomas Little Heath, an Englishman, attended Trinity College in Cambridge. He worked for English government from 1884 to 1926 as the Joint Permanent Secretary of the Treasury and Auditor of the Civil List. Heath's first publication was *Diophantos of Alexandria: a Study in the History of Greek Algebra*, which appeared when he was only twenty-four years of age, (1885) and a second, more scholarly edition appearing in 1910. His second work was *Apollonius of Perga, Treatise on Conic Sections*, which appeared in 1896. The third book published by him was

The Works of Archimedes in 1897. It is probable that the most important of Sir Thomas Heath's works is *The Thirteen Books of Euclid's Elements*, the object of our interest. This was published in three volumes in 1908 with a second edition appearing in 1926. In 1912 a new edition of the *Works of Archimedes* included the *Method of Archimedes* with commentaries as a supplement. Heath expanded his horizons in 1913 to include the history of Greek astronomy when he came out with *Aristarchus of Samos—The Ancient Copernicus*. After Heath's version of Euclid, probably his best known work is his *History of Greek Mathematics*.



Johan Ludvig Heiberg

(November 27, 1854–January 4, 1928)

Thomas Little Heath

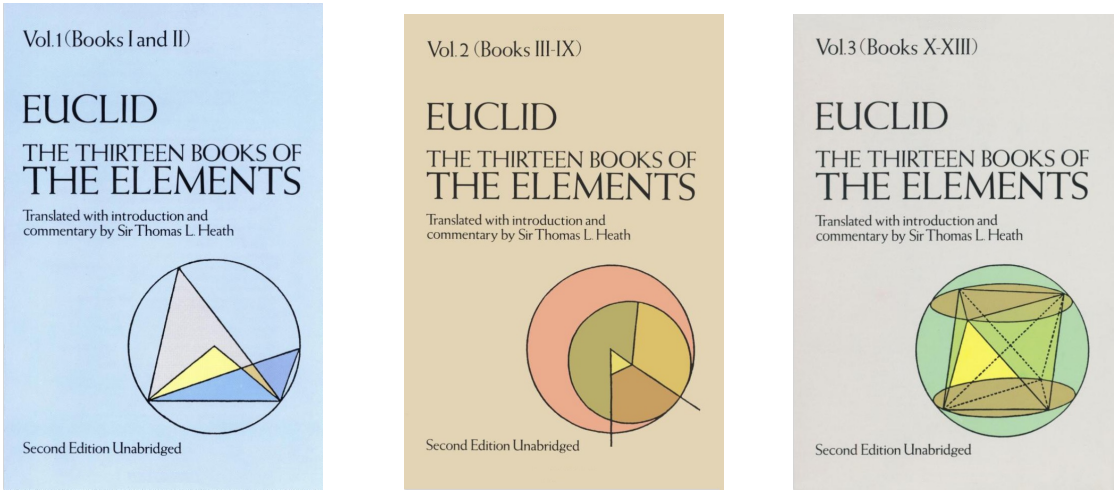
(October 5, 1861–March 16, 1940)

Images from Wikipedia webpage on Heiberg and MacTutor History of Mathematics Archive biography of Heath

Note. We have already referenced Heath's *The Thirteen Books of Euclid's Elements*. The full reference is

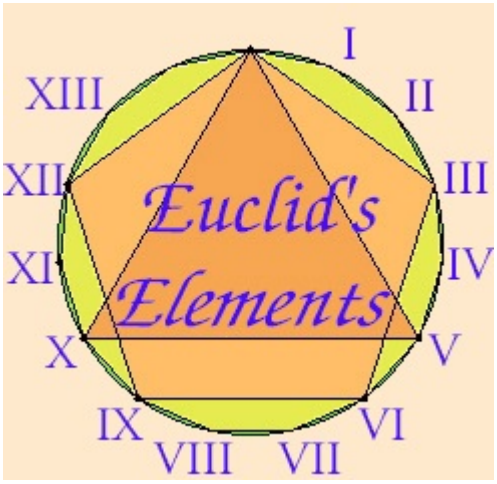
> Sir Thomas Heath, *The Thirteen Books of Euclids Elements, Translated From the Text of Heiberg with Introduction and Commentary*, 2nd Edition Revised with Additions, Cambridge University Press (1926).

The full title shows that Heiberg plays a pivotal role in the version of Eulcid's *Elements* that is currently widely available. As mentioned in our Part I. Classical Geometry, this version is still in print by Dover Publications.



Covers of the Dover editions of Heath's *Elements*.

Note. A very nice online presentation of Euclid's elements is David Joyce's (of Clark University) Euclid's *Elements* website:



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