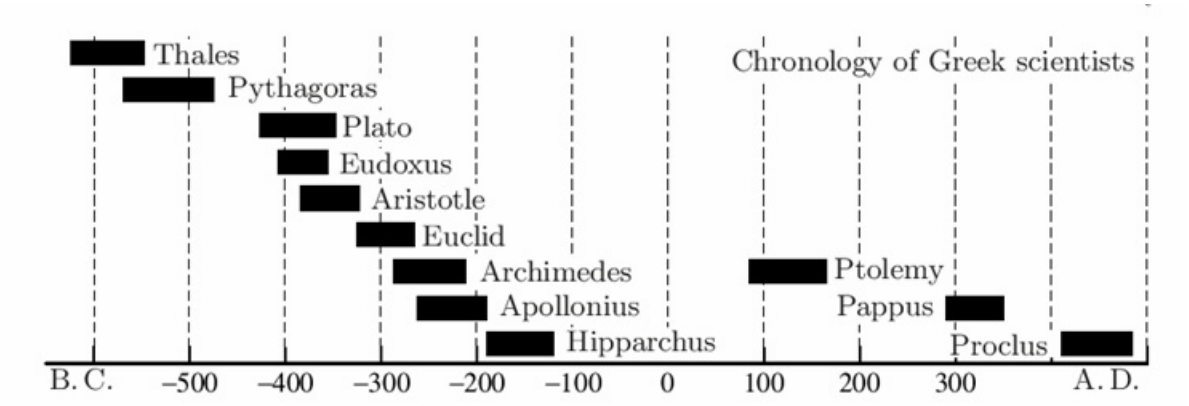
Part I. Classical Geometry

Note. Euclid lived around 300 BCE. Every introductory geometry class largely follows the outline given by Euclid in his *Elements of Geometry*. However, as we see in the figure below, Euclid was not the first geometer.



But little work survives from pre-Euclid times. Some think this is because Euclid's work was so organized that it simply replaced the earlier work. In *The Thirteen Books of Euclid's Elements* Translated from the Text of Heiberg with Introduction and Commentary by Sir Thomas Heath, Second Edition Revised with Additions, Cambridge University Press (1925) it is stated:

> "They are the more precious because the original works of the forerunners of Euclid, Archimedes and Apollonius are lost, having probably been discarded and forgotten almost immediately after the appearance of the masterpieces of that great trio." See page 29.

In Jason Socrates Bardi's *The Fifth Postulate—How Unraveling a Two-Thousand-Year-Old Mystery Unraveled the Universe*, Hoboken, NJ: John Wiley & Sons (2009), it is commented that:

> "Euclid eclipsed all previous mathematicians on the subject, and future generations would write little more than commentaries on his work." See page 50.

Some other quotes from Bardi concerning the *Elements* include:

> "The strange thing is, very little was new in the *Elements*... He pulled together the work of his predecessors, such as Eudoxus and Theaetetus. The tenth book of the *Elements* is almost entirely the work of Theaetetus." See page 50.
>
> "... when Isaac Newton wrote his Principia in the early 1680s, he copied its style from the Elements." See page 51.
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> "Some even claim that the Declaration of Independence owes its eternal form to the Greek method of proof exemplified by Euclid. 'We hold these truths to be self-evident,' it begins. And so it was for Euclid. His postulates were meant to be self-evident truths." See page 54.

Note. In Chapter 1, "Thales and Pythagoras," we consider pre-Euclid Greek geometry. We consider some work of Thales, similar figures, the construction of rational numbers, angles, and areas. We address the Pythagorean Theorem and the "Three Famous Problems of Greek Geometry." Constructibility and the three famous problems are motivation for much future work, some of it not resolved for two millenia!

Note. In Chapter 2, "The *Elements* of Euclid," we consider the definitions and postulates of Euclid. We consider several of the books of the *Elements* which cover geometry, including solid geometry and the Platonic solids.

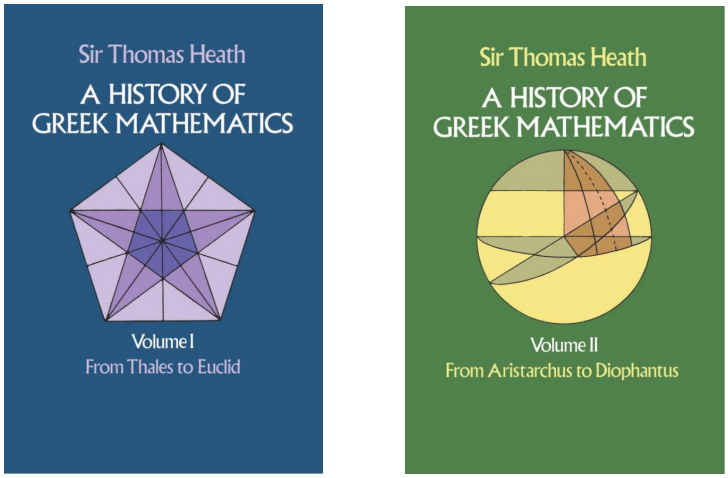
Note. Chapter 3 is on "Conic Sections." We consider the usual intersections of cones with planes, as studied in Pre-calculus or Calculus. Our attention is on the work of Apollonius and Pappus, along with a few others (including Archimedes, Kepler, and Euler). Chapters 1, 2, and 3 are of particular interest for the ETSU class Introduction to Modern Geometry (MATH 4157/5157).

Note. Chapter 4, "Further Results in Euclidean Geometry," contains results that the Greeks *could* have found with their methods, but apparently did not. Specific topics include the conchoid of Nicomedes, the Archimedean spiral, and several results concerning triangles and circles (including results from the 18th and 19th centuries by Euler, Gauss, and Steiner).

Note. Chapter 5, "Trigonometry," considers plane and spherical trigonometry. This concentrates on the work of Hipparchus and Ptolemy, both of whom were lead to this study due to their interest in astronomy. We consider the history of the trigonometric functions and state several trig identities which one encounters in high school trig and Pre-calculus 2 (MATH 1720). After considering spherical trigonometry, we mention the discoveries of Kepler and Newton concerning the motion of the planets.

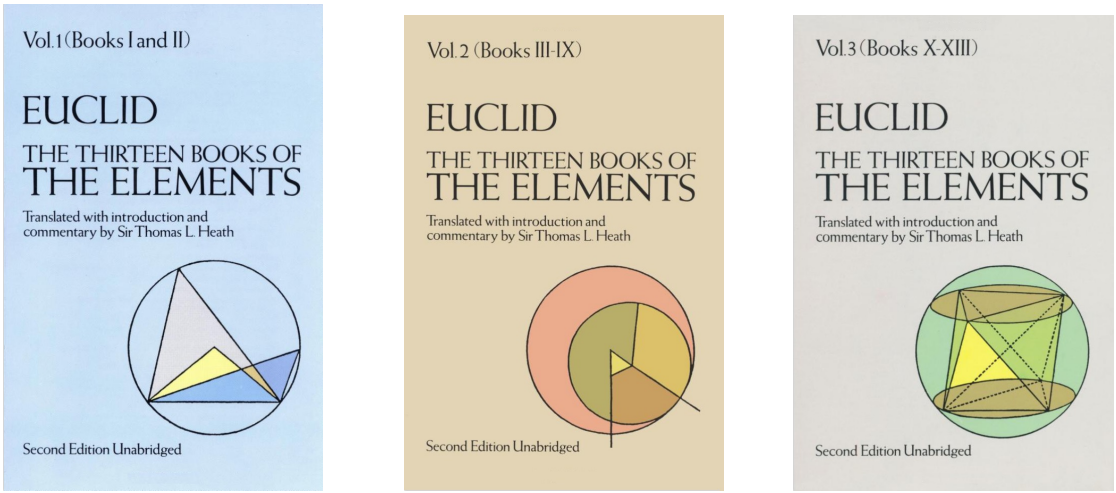
Note. Sir Thomas Heath (1861–1940) was involved in the translation of many of the works of classical Greek mathematics and science into English. His books are still in print, many available through Dover Publications. Of particular relevance in this chapter is his *A History of Greek Mathematics*, Oxford: Clarendon

Press (1921). Volume 1 of this work covers Thales to Euclid and Volume 2 covers Aristarchus to Diophantus.



Covers of the Dover editions of Heath's *History of Greek Mathematics*.

The definitive translation into English of Euclid's *Elements* is also due to Heath. These books are also still in print by Dover Publications. These works (and their extensive commentary by Heath; there are about 100 pages of historical comments before he starts Book I) will be particular useful when we cover Chapter 2, "The Elements of Euclid."



Covers of the Dover editions of Heath's *Elements*.