Preface.

Raphael's "The School of Athens" from RaphaelPaintings.org, painted 1509–1511

Note. Ostermann and Wanner start with the quote in Greek:

$$\grave{\alpha}\gamma\varepsilon\omega\mu\acute{\varepsilon}\tau\rho\eta\sigma\zeta \quad \mu\eta\delta\varepsilon\grave{\iota}\zeta \quad \varepsilon\grave{\iota}\sigma\acute{\iota}\tau\omega.$$

They translate this literally as "non-geometers don't enter." Sometimes this is translated as "Let no one unversed in geometry enter here" (see, for example, page 75 of Howard Eves' *An Introduction to the History of Mathematics*, 5th Edition, Saunders College Publishing [1983]; this is *the* standard book for an undergraduate course in the history of mathematics from the 1960s through the 1990s). This quote is supposed have been etched over the door to Plato's Academy (or "School") in Athens, Greece, though Ostermann and Wanner comment that this may be a legend. The quote also appears on the front piece of Copernicus' *De revolutionibus orbium coelestium* (or *On the Revolutions of the Heavenly Spheres*) in which he

introduces his theory of heliocentrism in 1543 (with the planets orbiting the Sun in circular orbits). Legend or history, this quote reflects the central role played by classical Euclidean geometry in the history of philosophy, astronomy, and mathematics. By the way, the central figures in Raphael's painting are Plato (left) and Aristotle (right).

Note. Raphael's painting, "The School of Athens" shown above, in fact includes an image of Euclid himself. A close-up of Euclid in the painting from ABC-People.com where he is portrayed using a compass is:

Note. Ostermann and Wanner declare that geometry "is the oldest branch of mathematics" (see page vii). This might be debatable, depending on what one interprets as "mathematics." Counting likely predates geometric ideas and evidence

of this exists in the fossil record (for example, in the form of the 20,000 year old Ishango bone, the fibula of a baboon, which includes notches cut into the bone which may count the days in a six-month lunar calendar). But the geometry from from over 2,000 years ago is still taught today, largely in a way similar to the way it was studied in the time of the ancient Greeks. The authors mention the Greek mathematicians Thales, Pythagoras, Euclid, Apollonius, Archimedes, Ptolemy, and Pappus. We will consider some of the work of each of these.

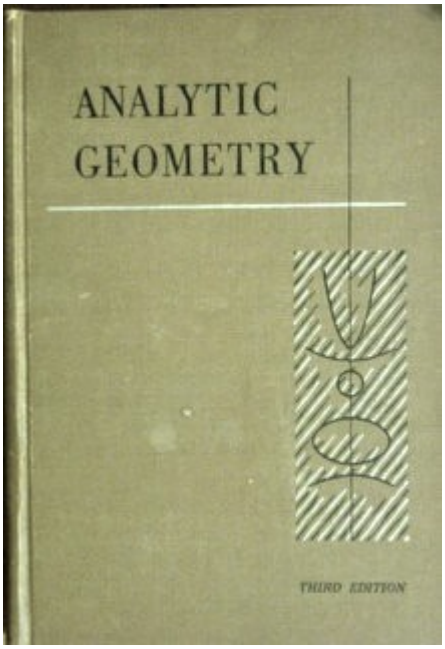
Note. The authors quote Niels Henrik Abel (a character you will meet in Introduction to Modern Algebra [MATH 4127/5127]; see my online notes for this class on A Students Question: Why The Hell Am I In This Class? which gives some math history and places Abel in his role in the history of algebra and the theory of equations) as saying: "I learned from the masters and not from the pupils." Their point is that we can learn by reading the original works of the pioneers of an area. This is an idea with which your humble instructor largely disagrees! Often times the original work is cleaned up by future generations; there are lots of smart people that build on the original work, clean it up, or place it in a more general setting where a broader understanding is possible. For example, I challenge you to read Newton's *Treatise of the Methods of Series and Fluxions* (1671; also mentioned in the Preface) and walk away with clear understanding of calculus. However, one of the areas where reading the original work is beneficial is in geometry! Euclid's *Elements of Geometry* can be read today with benefit, though the wording is clunky be modern standards. Some of the ideas in the geometry of the ancient Greeks were ultimately address two thousand years later, putting some relatively modern

mathematics in historical context. In particular, the resolution of the three classical compass and straight-edge problems (which we consider in Section 1.8 "Three Famous Problems of Greek Geometry") was given in the setting of field theory and constructible numbers in the 1800s. See my online presentations on "Compass and Straight Edge Constructions" which are available in PowerPoint, with a transcript, and in video. The video is also on TouTube.

Note. The Pythagorean philosophy gave mystical powers to (positive) whole numbers and the Pythagorean school taught the four central topics of arithmetics (the theory of numbers), geometry, music, and astronomy (or "the spherics"). These four subjects came to make up the *quadrivium* (the "four ways"), which combined with the *trivium* (grammar, logic, and rhetoric) to make up the *septem artes liberales* (the "seven liberal arts") that were the topics taught in medieval times (about the 5th to the late 15th centuries) as part of a liberal arts degree (a *baccalaureus artium*). See page 48 of Howard Eves' *An Introduction to the History of Mathematics*, 5th Edition, mentioned above.

Note. Ostermann and Wanner decry the absence of geometry in a contemporary undergraduate education. I know of no college level freshman geometry class in any university! The standard freshman mathematics classes (preceding calculus) are "college algebra" and trigonometry (at ETSU, these are Precalculus 1—Algebra [MATH 1710] and Precalculus 2—Trigonometry [MATH 1720], respectively). Of course an introductory statistics class often plays a role as a general education class

(this is a new trend, becoming widespread in the last 25 or 30 years). In fact, my father took College Geometry in the 1950s as a freshman (along with an algebra class):



Analytic Geometry, 3rd Edition, by W.A. Wilson and J.I. Tracey
(D.C. Heath and Company, 1949)

At ETSU we have this course, "Introduction to Modern Geometry" (MATH 4157/5157) at the senior/graduate level (these notes were prepared with Introduction to Modern Geometry in mind, though they could also be used in "History of Mathematics" [MATH 3040]). A related course is "Introduction to Topology" (MATH 4357/5357), also senior/graduate level, which covers geometric ideas but requires a significant background in pure math; "Analysis 1" (MATH 4217/5217) would be a reasonable prerequisite. I have online notes for "Introduction to Topology", as well as online notes for "Analysis 1" and online notes for "Analysis 2". At the graduate-only level, ETSU offers a class in "Differential Geometry" (MATH 5310); this deals with curvature of manifolds and finds application in Einstein's theory

of general relativity. I have online notes for "Differential Geometry" and alternate (more technical) notes for the class based on the book *Tensor Geometry: The Geometric Viewpoint and its Uses* by Dodson and Poston (Springer-Verlag, 1991). So geometry courses are alive and well at ETSU but, regrettably, we only have upper-level options.

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