Chapter 1. Thales and Pythagoras
Study Guide

The following is a brief list of topics covered in Chapter 1 of Ostermann and Wanner's *Geometry by Its History*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Section 1.1. Thales' Theorem.
Thales of Miletus, Thale's five listed results, Thales' Intercept Theorem (Theorem 1.1), the idea that Thales' results hold only for rational proportions due to the method of proof.

Section 1.2. Similar Figures.
Similar figures with similarity center O, constructing rational lengths.

Section 1.3. Properties of Angles.
Parallel lines, transversal, alternate interior angles, corresponding angles, opposite angles, the sum of the angles of a triangle (Theorem 1.2/Euclid I.32), exterior angle, straight angle, the exterior angle is the sum of the non-adjacent interior angles (Corollary 3.1), central angle (in a circle)on an arc, inscribed angle (in a circle) on an arc, a central angle of a circle is twice any inscribed angle on the same arc (Theorem I.4), if AB is the diameter and C is a point (other than A or B on the circle, then the angle ACB is a right angle (Theorem 1.5/Euclid III.31).

Section 1.4. The Regular Pentagon.
The golden ratio Φ and its relationship to a pentagon, the Parthenon and the golden ratio (and the skepticism about the relationship), $\Phi = 1 + 1/\Phi$, irrationals (incommensurables) and Pythagoras.

Section 1.5. The Computation of Areas.
Area of a rectangle, area of a triangle, area of a trapezium, the Rhind papyrus, Egyptian decimal system, similar triangles, a similar triangle with q times longer sides has q^2 times larger area (Theorem 1.6/Euclid VI.19).

Section 1.6. A Remarkable Babylonian Document.
Babylonian tablet YBC 7289, translation of the Babylonian numbers into Arabic and their use in estimating $\sqrt{2}$. the Babylonians and the Pythagorean Theorem.

Section 1.7. The Pythagorean Theorem.

The Chinese, Indian, and Arabic proofs of the Pythagorean Theorem (Figure 1.15), Euclid's proof of the Pythagorean Theorem (Euclid I.47; Figure 1.19), Vatican Manuscript Number 190, Fibonacci's proof of the Pythagorean Theorem (Figure 1.20), Pythagoras' proof (Heath's commentary from *A History of Greek Mathematics*, and Figure 1.21), the radii of the incircle and circumcircle of a regular polygon with sides of length 1 (Table 1.1).

Section 1.8. Three Famous Problems of Greek Geometry.

Euclidean tools and Euclid's approach to proof in the *Elements*, quadratic space, the Three Famous Problems: Doubling the Cube; Squaring the Circle; Trisect an Angle, the role of $\sqrt[3]{2}$ in doubling the cube, the role of $\sqrt{\pi}$ is squaring the circle, the role of a 20° angle in trisecting an angle, the definition of π, Archimedes' proof that the area of a circle is $A = \pi r^2$ (in his *Measurement of a Circle*), the method of exhaustion, the use of field theory and constructible numbers to shwo the impossibility of the three famous problems, Hippocrates of Chios and the lunes of Hippocrates.

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