Chapter 2. The Elements of Euclid
Study Guide

The following is a brief list of topics covered in Chapter 2 of Ostermann and Wanner's *Geometry by Its History*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Chapter 2. The Elements of Euclid.
13 books, Euclid the person, areas covered by the books, other works by Euclid, Proclus' commentaries, how the *Elements* was translated and reached us, Johan Ludvig Heiberg, Sir Thomas Little Heath, David Joyce's online copy of the *Elements*.

Section 2.1. Book I.
Euclid's definitions and postulates, homogeneity and isotropy of Euclidean geometry, the Parallel Postulate (Postulate 5), Euclid's common notions, Proposition I.1 and criticism, Proposition I.2 and "compass carry," Proposition I.4 (side-angle-side) and criticism, Proposition I.5 (equal sides in a triangle are opposite equal angles) and the *pons asinorum* story, continuity in geometry using The Ruler Postulate and the Axiom of Completeness, Proposition I.7, indirect proof/proof by contradiction/*reductio ad absurdum*, Proposition I.8 (equal sides in two triangles imply equal angles), transformational geometry, the underlying compass and straightedge construction approach by Euclid, invariability of figures, local geometry, right angle, Propositions I.13 to I.16, angle-side-angle, angle-angle-side, the Triangle Inequality, Euclid's definition of parallel lines (Definition 23), Propositions I.27 and I.28, absolute or neutral geometry, non-Euclidean geometry (hyperbolic geometry and elliptic geometry), Proposition I.29 (the first proposition to use the Parallel Postulate), summit angles of a Saccheri quadrilateral, the Gauss/Lobachevsky/Bolyai controversy over the discovery/invention of hyperbolic geometry, Propositions I.30 and I.31 (concerning parallel lines), Playfair's Axiom as a replacement for the Parallel Postulate, Book II and geometrical algebra, geometric proofs of algebraic formulae (Proposition II.4 and Figure 2.14).

Section 2.2. Book III. Properties of Circles and Angles.
The Central Angle Theorem (Proposition III.20), Proposition III.31 (diameters of circles determine right angles), Proposition III.21, Proposition III.22 and its proof (opposite angles in a quadrilateral inscribed in a circle), Proposition III.35 (intersecting chords in a circle), Proposition III.36 and its corollary and their proofs, power of a point with respect to a circle/symmetric with respect to a circle.

Section 2.3. Books V and VI. Real Numbers and Thales' Theorem.
Commensurables/incommensurables, magnitudes, Proposition VI.1, Proposition VI.2 (Thales' Theorem), Proposition VI.3, constructible numbers.

Section 2.4. Books VII and IX. Number Theory.
Unit, number, part (divisor), multiple, geometric interpretations of sums and differences, Proposition VII.2 (The Euclidean Algorithm), Proposition IX.20 (there are an infinite number of primes), Proposition X.1 and the idea of am "ε proof," Proposition X.2 (classification of incommensurables), application of Proposition X.2 to show Φ and $\sqrt{2}$ are irrational, Pythagorean triples.

Section 2.5. Book XI. Spatial Geometry and Solids.
Solid, face, a line at a right angle to a plane, parallel planes, pyramid, prism, cone, cylinder, Platonic solids, tetrahedron, cube, octahedron, dodecahedron, icosahedron, the association of the Platonic solids with the classical Greek elements, duality of Platonic solids, parallelepiped, right-angled parallelepiped.

Section 2.6. Book XII. Areas and Volumes of Circles, Pyramids, Cones.
Proposition XII.1 (areas of similar polygons inscribed in circles), Proposition XII.2 (ratio of areas of circles equal to the square of the ratios of their radii), π, Archimedes approximation of π in *Measurement of a Circle*, volumes of pyramids, Proposition XII.17 (ratio of volumes of spheres equal to the cube of the ratios of their radii), Archimedes formula for the volume of a sphere in *On Conoids and Spheroids*, Euclid's constructions of the Platonic solids, Euclid's remark that there are only five Platonic solids, comments concerning Euclid's assumptions of convexity and congruent solid angles (as illustrated by the stellated dodecahedron and the gyroelongated square bipyramid), the (false) idea that Euclid's *Elements* was done to ultimately show the existence and uniqueness of the five Platonic solids (motived maybe by the comment of Proclus).

Section 2.7. Epilogue.
Legendre's 1794 *Elements of Geometry*, David Hilbert's modern axiomatic approach to Euclidean geometry and his 1899 *The Foundations of Geometry*, George Birkhoff's 1932 paper "A Set of Postulates for Plane Geometry (Based on Scale and Protractors)," tha axiomatic approach taken in Introduction to Modern Geometry (MATH 4157/5157).

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