Chapter 3. Conic Sections

Study Guide

The following is a brief list of topics covered in Chapter 3 of Ostermann and Wanner's *Geometry by Its History*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Chapter 3. Conic Sections.
Apollonius of Perga and his *Treatise on Conic Sections*, Menaechmus and the doubling of the cube, Eutocius' description of Menaechmus' doubling the cube (Note 3.A), the absence of an influence of Apollonius on modern presentations of conic sections.

Section 3.1. The Parabola.
Pappus of Alexandria's *Mathematical Collection*, the directrix and focus definition of a parabola, the axis and vertex and equation of a prarbola (Note 3.1.A), latus rectum, semi latus rectum, Apollonius' reason for using the term "parabola" and the comparison of the area of a square and a rectangle (Note 3.1.B), agreement of Menaechmus' and Pappus' definitions (Theorem 3.1; Apollonius' Proposition I.11), Germinal Dandelin and Dandelin spheres, compass and straight edge construction of a tangent to a parabola (Note 3.1.C), the use of calculus to show the reflective property of a parabola ("Exercise 11.6.81").

Section 3.2. The Ellipse.
The directrix and focus definition of an ellipse, eccentricity of an ellipse, the equation of an ellipse (equation (3.3)), Apollonius' reason for using the term "ellipse" and the comparison of the area of a square and a rectangle (Note 3.2.A), agreement of Menaechmus' and Pappus' definitions (Theorem 3.2.A; Apollonius' Proposition III.52), the definition of an ellipse in terms of the sum of distances from two fixed points (foci), the agreement of the three definitions of an ellipse (of Meneachmus, Apollonius, and Pappus; Note 3.2.B), compass and straight edge constructions of tangents to an ellipse and Apollonius' Theorem III.48, using calculus to show the reflective properties of ellipses (Note 3.2.C), minor axis, semi-minor axis, major axis, semi-major axis, semi-latus rectum, Archimedes and the area of an ellipse (based on the method of exhaustion), Archimedes approximates π, Proclus' construction of an ellipse "with a stick," Proclus construction of an ellipse using two concentric circles (Notes 3.2.D) and its justification, conjugate diameters of an ellipse.

Section 3.3. The Hyperbola.
Definition of a hyperbola in terms of distances from two fixed points (or foci; this is the "Apollonius

Property" of a hyperbola), agreement of Menaechmus' and the distance from two foci definitions of a hyperbola (Theorem 3.3.A; Apollonius' Proposition III.51), the directrix and bocus definition of a branch of a hyperbola, eccentricity, agreement between the three definitions of a hyperbola (Note 3.3.A), vertex, equation of a hyperbola (equation (3.11)), Apollonius' reason for using the term "hyperbola" and the comparison of the area of a square and a rectangle (Note 3.3.B), using calculus to show the reflective properties of hyperbolas (Note 3.3.C), asymptotes of a hyperbola, the modern interpretation of an asymptote versus Apollonius' interpretation.

Section 3.4. The Area of a Parabola.

Archimedes and his *Quadrature of the Parabola*, Thābit ibn Quarra's Riemann integral style approach to the area question, Archimedes' *The Method*, Archimedes approach of summing areas of triangles and the proof of Theorem 3.4.A, Plutarch's story of Archimedes' death, Archimedes' other surviving works, Archimedes' *Measurement of a Circle* and his approximation of π, Archimedes' *The Method* and his near-invention of calculus, the story of *The Method* and the palimpsest containing it.

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