# 1.5. Conclusion

**Note.** We considered in Section 1.1 the historical origins of projective geometry based on the theory of perspective as developed by Renaissance artists. In Section 1.2 we introduced the constituent parts of the perspective transformation and argued that is is conceptual similar to the approach used by Albrecht Dürer. We used it to address perspective transformations of parallel lines, and argued the existence of the vanishing line $v$. Based on Example 1.2.1 (in which we considered the projection of a circle), we stated the formulae used in a perspective transformation (see Note 1.2.G), but left the verification of these formulae to exercises. In Section 1.3 we followed the perspective transformation with a $90°$ rotation of the picture plane onto the object plane (called "rabattement"), defining the plane perspective transformation. Invariant points were determined, and a plane perspective was uniquely determined from its center $O$, axis $l$, and a pair of points $(G, G')$ where $G$ is not invariant and $G'$ is its image under the transformation (in Theorem 1.3.B). The vanishing line $v$ of a plane perspective was defined. The image of a point under a plane perspective was described in terms the axis $l$, center $O$, and vanishing line $v$ Iin Theorem 1.3.1). We saw that a plane perspective is uniquely determined from its center $O$, vanishing line $v$, and a pair of points $(G, G')$ where $G$ is not invariant and $G'$ is its image under the transformation (in Corollary 1.3.B). In Section 1.4, we considered images of several geometric figures under the plane perspective transformation. We mapped a quadrilateral to a parallelogram, then to a rectangle, and then to a square. We considered a circle and its images under the plane perspective and showed how the type of image (an ellipse, parabola, or hyperbola) can be determined by the circles relationship to the vanishing line $v$.

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