

Chapter 1. The Elements of Perspective

Study Guide

The following is a brief list of topics covered in Chapter 1 of C. R. Wylie' *An Introduction to Projective Geometry* (McGraw-Hill, 1970). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs, constructions, and examples given in class, and the homework problems.

Section 1.1. Introduction.

Perspective of painters of ancient Greece and Rome, an intuitive theory of perspective (Duccio di Buoninsegna and Giotto di Bondone), Ambrogio Lorenzetti, Filippo Brunelleschi and his work (in particular, the dome of the Santa Maria del Fiore in Florence, Italy), linear perspective, Leon Alberti and his *De pictura*, Piero della Francesca and his *Divine Proportions*, Luca Pacioli, Leonardo da Vinci and his *A Treatise on Painting*, Albrecht Dürer and his *Four Books on Measurement*, Raffaello Sanzio da Urbino (“Raphael”) and his “The School of Athens,” projection, section.

Section 1.2. The Elements of Perspective.

Viewing point, picture plane (image plane, ρ), object plane σ , projecting line (line of sight), image, perspective transformation, principal vanishing point V , vanishing line (horizon line, v), similarities between Dürer's approach and the perspective transformation, parallel lines in the object plane perpendicular to the picture plane (Note 1.2.B and Theorem 1.2.1), parallel lines in the object plane which are not perpendicular to the picture plane (Note 1.2.C), lines in the object plane which are parallel to the line of intersection of the object plane and the picture plane (Note 1.2.D), Theorem 1.2.2 (a summary of Notes 1.2.B, 1.2.C, and 1.2.D), Example 1.2.1 and the quantifying of the perspective transformation, auxiliary coordinates (X, Y) and (X', Z') , points on the vanishing line as limit points of images of lines in the object plane (Example 1.2.1(b) and Exercise 1.2.8(a)), perspective transformation of a circle (Example 1.2.1(c), Exercise 1.2.1, and Note 1.2.E), the vanishing line of the “plane perspective (Exercise 1.2.8(b)), the perspective transformation of the whole object plane (Note 1.2.F), quantitative relationship between auxiliary coordinates (X, Y) and (X', Z') (Note 1.2.G), quantitative proofs of Theorems 1.2.1 and 1.2.2 (Exercise 1.2.B), perspective transformations map conic sections to conic sections (Exercise 1.2.C(a)).

Section 1.3. Plane Perspective.

Rabattement, invariant line l , preimage, invariant point O , image of point $P \neq O$ (Note 1.3.C and Figure 1.8(a)), plane perspective, axis l , center O , image of a line in the object plane through center O (Theorem 1.3.A), unique determination of a plane perspective using the axis (Theorem 1.3.B), points without an image under plane perspective (Note 1.3.D and modified Figure 1.8(b)), vanishing line of the plane perspective, the image of a line under the plane perspective transformation (Theorem 1.3.1 and Corollary 1.3.1), the image of a point under the plane perspective transformation in terms of the center O and the vanishing line v (Theorem 1.3.2), unique determination of a plane perspective using the vanishing line (Corollary 1.3.B).

Section 1.4. Plane Constructions.

Mapping a quadrilateral to a parallelogram with a plane perspective transformation (Note 1.4.A and Figure 1.9), mapping a quadrilateral to a rectangle (Note 1.4.B and Figure 1.10), mapping a quadrilateral to a square (Note 1.4.C and Figure 1.11), circles that are mapped to ellipses (Note 1.4.E and Figure 1.12), circles that are mapped to hyperbolas (Note 1.4.F and Figure 1.13), circles that are mapped to parabolas (Note 1.4.G and Figure 1.14), intuitive interpretation of the plane perspective transformation and mapping of points an line “to infinity” (Note 1.4.H).

Section 1.5. Conclusion.

A summary of the content of Chapter 1.

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