Chapter 1. The Axiomatic Method

Note. This chapter gives a nice conversational introduction to some foundational and philosophical mathematical ideas. Axiomatic systems are described and the consistency, independence, and completeness of an axiomatic system are discussed.

1.1. Introduction

Note. The word "geometry" comes from the Greek words *ge*, meaning "earth," and *metrien*, meaning "measure." Historically, geometry initially consisted of a hodge-podge collection of methods for calculating areas and volumes. The Babylonians and Egyptians used geometry and such calculations in the construction of tombs, temples, and other structures. Some of the techniques were precise and some were only approximations. There was no sense of generalizations nor of proof. As Wylie states on page 1: "... for the first two thousand yours or so of its existence, geometry was a body of empirical knowledge obtained inductively from a consideration of many special cases and completely unsupported by anything resembling logical proof."

Note. In the west, the Greeks took the geometry of the Egyptians, abstracted it, and created a deductive science of geometry. Thales of Miletus (624 BCE–547 BCE) is often considered the first geometer. Another early geometer is Pythagoras of Samos (about 570 BCE–495 BCE), who formed a secretive, mystical school in Crotone (or Kroton) Italy. The Pythagorean school assigned mystical powers

to numbers and certain geometric figures. Euclid lived around 300 BCE. In his *Elements of Geometry* he collected together the known results of geometry and organized them into a logical chain of results, starting with definitions and axioms. Euclid's book was so thorough that it seems to have replaced earlier works, leaving the historical record of the genesis of Greek geometry sparse. For more details, see my online notes on History of Geometry Part I (see Chapters 1 and 2, in particular).

Note. Euclid starts with definitions, postulates, and axioms. This approach is called the *axiomatic method*. We discuss this method in the remainder of Chapter 1. We will develop Euclidean geometry axiomatically in Chapter 2 (in an updated way that adds a bit more rigor to Euclid's approach). It is safe to say that Euclid's *Elements* is the most influential math book ever! To this day, upper-level mathematics text books are laid out much in the same axiom/definition/theorem/proof style.

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