2.10. The Parallel Postulate

Note. In this section, we

Definition 2.10.1. Two coplanar lines which do not intersect are *parallel lines* and each is said to be *parallel* to the other. We denote the fact that p and q are parallel lines as $p \| q$. Two noncoplanar lines are *skew lines* and are said to be it skew to each other.

Definition 2.9.2. A *transversal* of two coplanar lines is a line which intersects the union of the two lines in two points.

Definition. Let p and q be two coplanar lines and t a transversal of the lines. If P and Q are the points in which t intersects p and q, respectively (see Figure 2.27). If A and B are points on line p on opposite sides of P, and if C and D are points on line q on the same side of $\overleftrightarrow{PQ}$ as A and B, respectively, then $\angle APQ, \angle DQP$ and $\angle BPQ, \angle CQP$ are pairs of *alternate interior angles*, and $\angle APQ, \angle CQP$ and $\angle BPQ, \angle DQP$ are pairs of *consecutive interior angles.* If α and β are a pair of alternate interior angles then either angle and the angle vertical to the other are *corresponding angles.*

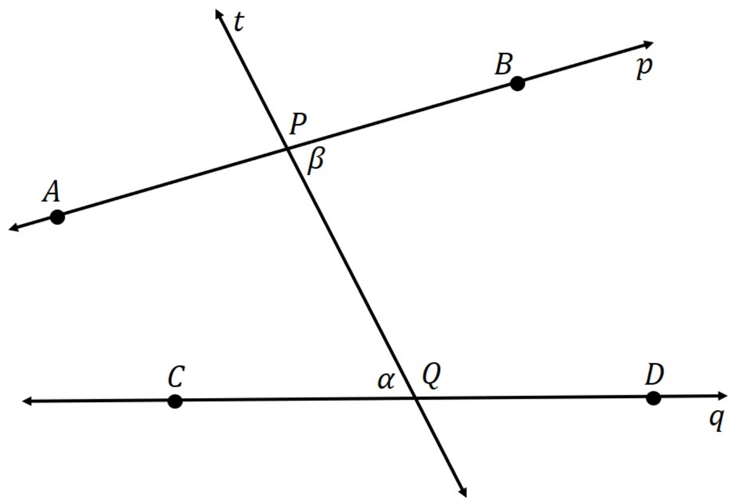

Figure 2.27

Theorem 2.10.1. If a pair of alternate interior angles formed by two coplanar lines and a transversal are congruent, then the lines are parallel.

Corollary 2.10.1. Two coplanar lines which are perpendicular to the same line are parallel.

Theorem 2.10.2. In the plane determined by a given point and a given line not containing the point, then there exists a line which passes through the given point and is parallel to the given line.

Postulate 17. The Parallel Postulate. There is at most one line parallel to a given line through a given point not on that line.

Theorem 2.10.3. If two lines are parallel, then the alternate interior angles determined by these lines and any transversal are congruent.

Theorem 2.10.4. A transversal which is perpendicular to one of two parallel lines is perpendicular to the other also.

Theorem 2.9.5. A line lies in the plane of two parallel lines and intersects one of the lines in a single point intersects the other in a single point also.

Theorem 2.10.6. I each of two coplanar lines is parallel to a third line, they are parallel to each other.

Definition 2.10.3. Two segments are *parallel segments* if and only if the lines which they determine are parallel.

Definition 2.10.4. Two rays, $\overrightarrow{AB}$ and $\overrightarrow{CD}$, are *parallel rays* if and only if

(1) the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, and

(2) B and D lie on the same side of $\overleftrightarrow{AC}$.

Definition 2.10.5. Two rays, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are *antiparallel rays* if and only if:

(1) the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, and

(2) B and D lie on opposite sides of $\overleftrightarrow{AC}$.

Definition 2.10.6. A quadrilateral each of whose sides is parallel to the side opposite it is a *parallelogram.*

Theorem 2.10.7. Opposite sides of any parallelogram are congruent.

Theorem 2.10.8. If two sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram

Theorem 2.10.9. The diagnonals of a parallelogram bisect each other.

Theorem 2.10.10. If three or more parallel lines determine congruent segments on one transversal, then they determine congruent segments on every transversal.

Revised: 11/19/2021