Section 1.6. Finite Sample Spaces

Note. In this brief section we let the sample space be $S = \{s_1, s_2, \ldots, s_n\}$ where $\Pr(\{s_i\}) = p_i$. So we must have $p_i \geq 0$ for $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} p_i = 1$. By Theorem 1.5.2, Finite Additivity, the probability of any event is the sum of the probabilities of the outcomes in the event.

Definition. A sample space $S = \{s_1, s_2, \ldots, s_n\}$ for which $\Pr(\{s_i\}) = p_i = 1/n$ for $i = 1, 2, \ldots, n$ is a *simple sample space*.

Example 1.6.3. Tossing Coins. Suppose three fair coins are tossed simultaneously. The sample space is as given in Example 1.4.1. We take each of the eight outcomes to have the same probability. The probability of the event that there are exactly two heads is

$$\Pr(\{THH, HTH, HHT\}) = \Pr(\{s_2, s_3, s_4\}) = 3/8.$$

Example 1.6.5. Rolling Two Dice. Consider the experiment of rolling two balanced dice. The 36 outcomes (which we take to have equal probability) are:

$$\begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6). \end{array}$$

So each outcome has probability 1/36. We let P_i denote the probability that the sum of the two dice is i. We then have (as can be seen from the sample space):

$$\begin{array}{ll} P_2 = P_{12} = 1/36 & P_5 = P_9 = 4/36 \\ P_3 = P_{11} = 2/36 & P_6 = P_8 = 5/36 \\ P_4 = P_{10} = 3/36 & P_7 = 6/36. \end{array}$$

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