

Mathematical Statistics 1, Chapter 1

Study Guide

Chapter 1. Introduction to Probability

The following is a *brief* list of topics covered in Chapter 1 of DeGroot and Schervish's *Probability and Statistics*, 4th edition, and the associated supplements. This list is not meant to be comprehensive, but only gives a list of several important topics.

1.1. The History of Probability.

Blaise Pascal, Pierre Fermat, Girolamo Cardano, Galileo Galilei, measure theory based probability.

1.2. Interpretations of Probability.

The frequency interpretation, the classical interpretation, the subjective interpretation of probability.

1.3. Experiments and Events.

Experiment, outcome, event.

1.4. Set Theory.

Sample space, three conditions on the set of events, subset, superset, equal sets, transitivity of set inclusion (Theorem 1.4.1), empty set \emptyset , countable and uncountable sets, complement of a set, union of sets, σ -algebra of sets, associative property of unions (Theorem 1.4.6), intersection of sets, associativity of intersections of sets (Theorem 1.4.8), disjoint/mutually exclusive, De Morgan's Laws (Theorem 1.4.9), distributive properties of unions and intersections (Theorem 1.4.10), partitioning a set (Theorem 1.4.11).

1.5. The Definition of Probability.

The axioms of probability (Axiom 1. Axiom of Non-Negativity, Axiom 2. Axiom of Total Probability, and Axiom 3. Axiom of Countable Additivity), probability measure/probability, finite additivity (Theorem 1.5.2), probability of the complement (Theorem 1.5.3), monotonicity (Theorem 1.5.4), probability of two events (Theorem 1.5.7), Bonferroni Inequality (Theorem 1.5.8), finite/countable subadditivity.

1.6. Finite Sample Spaces.

Simple sample space, tossing coins, rolling two dice.

1.7. Counting Methods.

Multiplication Rule, an experiment with k parts (Theorem 1.7.2), factorial, permutations, sampling with/without replacement, Obtaining Different Numbers (Example 1.7.11), the Birthday Problem, Stirling's Formula (Theorem 1.7.5).

1.8. Combinatorial Methods.

Combinations and Theorem 1.8.1, the Binomial Theorem (Theorem 1.8.2), sampling without replacement (Example 1.8.8), the Tennis Tournament.

1.9. Multinomial Coefficients.

Choosing n elements from a set of objects in k different groups, multinomial coefficients, the Multinomial Theorem (Theorem 1.9.1), Playing Cards (Example 1.9.4).

1.10. The Probability of a Union of Events.

The probability of the union of three events (Theorem 1.10.1), the probability of the union of n events (Theorem 1.10.2), the Matching Problem.

1.11. Statistical Swindles.

A Perfect Forecast, the Guaranteed Winner, Improving Your Lottery Chances.