Introduction to Knot Theory

Chapter 5. Algebraic Techniques

5.4. Equations in Groups and the Group of a Knot—Proofs of Theorems

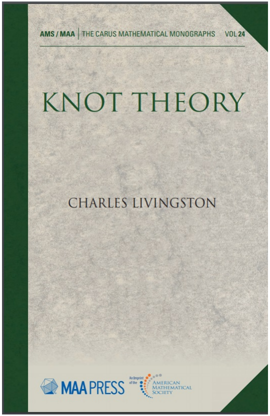

Table of contents

Example 5.4.A

Example 5.4.A. Let G be a group and let arcs of the oriented graph in Figure 5.8 be labeled $x, y, z \in G$, as given. Then the other given labels on arcs are as presented.

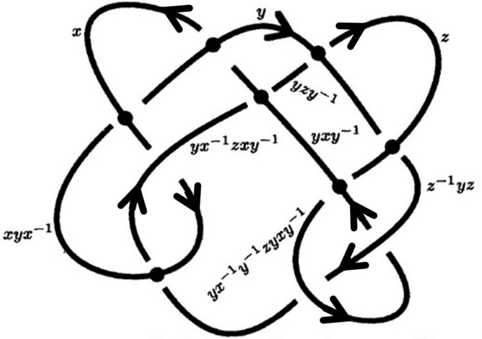

Figure 5.8

Solution. We go through the crossings one at a time.

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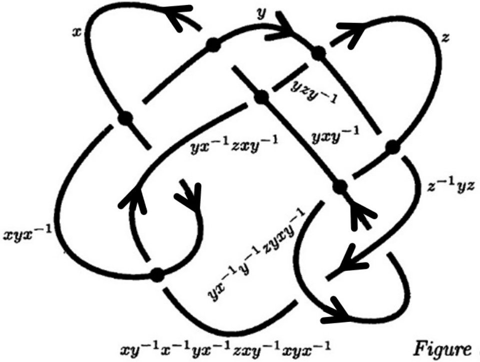


Figure 5.8

Solution. We go through the crossings one at a time.

Example 5.4.A (continued 1)

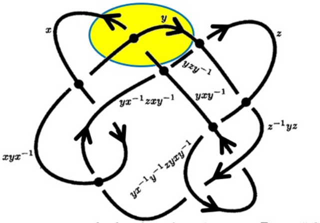
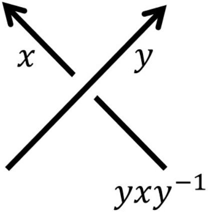


Figure 5.8



Solution (continued). This is a right-handed crossing, so we need $(y)(z)(y)^{-1} = yzy^{-1}$ and hence the consistency condition is satisfied.

Example 5.4.A (continued 2)

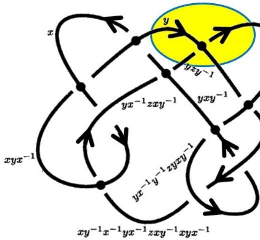
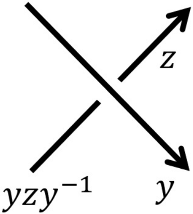


Figure 5.8



Solution (continued). This is a right-handed crossing, so we need $(y)(x)(y)^{-1} = yxy^{-1}$ and hence the consistency condition is satisfied.

Example 5.4.A (continued 3)

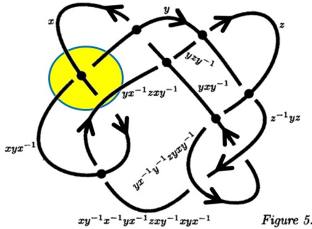
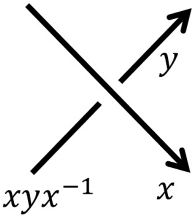
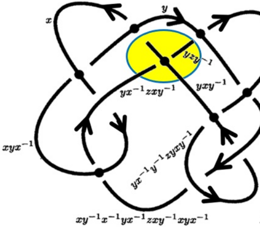


Figure 5.8



Solution (continued). This is a right-handed crossing, so we need $(x)(y)(x)^{-1} = xyx^{-1}$ and hence the consistency condition is satisfied.

Example 5.4.A (continued 4)



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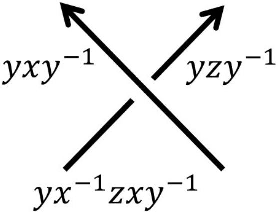
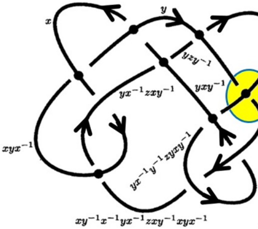


Solution (continued). This is a left-handed crossing, so we need $(yxy^{-1})(yx^{-1}zxy^{-1})(yxy{-}1)^{-1} = yzy^{-1}$. We have $yxy^{-1}yx^{-1}zxy^{-1}yx^{-1}y^{-1} = y(x(y^{-1}y)x^{-1})z(x(y^{-1}y)x^{-1})y^{-1} = yzy^{-1}$, and hence the consistency condition is satisfied.

Example 5.4.A (continued 5)



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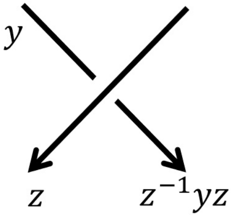


Solution (continued). This is a right-handed crossing, so we need $(z)(z^{-1}yz)(z)^{-1} = y$ and hence the consistency condition is satisfied.

Example 5.4.A (continued 6)

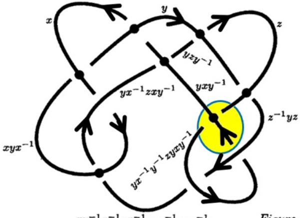
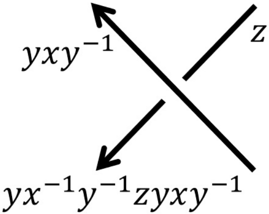
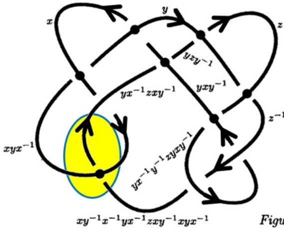


Figure 5.8



Solution (continued). This is a right-handed crossing, so we need $(yxy^{-1})(yx^{-1}y^{-1}zyxy^{-1})(yxy^{-1})^{-1} = z$. We have $yxy^{-1}yx^{-1}y^{-1}zyxy^{-1}yx^{-1}y^{-1} = (y(x(y^{-1}y)x^{-1})y^{-1})z(y(x(y^{-1}y)x^{-1})y^{-1}) = z$, and hence the consistency condition is satisfied.

Example 5.4.A (continued 7)



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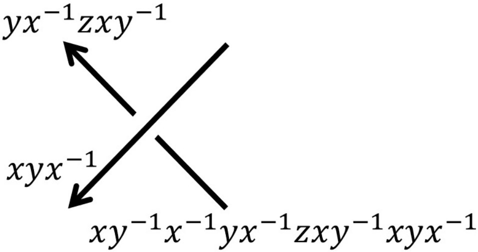

Solution (continued). This is a left-handed crossing, so we need $(xyx^{-1})(xy^{-1}x^{-1}yx^{-1}zxy^{-1}xyx^{-1})(xyx^{-1})^{-1} = yx^{-1}zxy^{-1}$. We have

$$xyx^{-1}xy^{-1}x^{-1}yx^{-1}zxy^{-1}xyx^{-1}xy^{-1}x^{-1}$$

$$= (x(y(x^{-1}x)y^{-1})x^{-1})yx^{-1}zxy^{-1}(x(y(x^{-1}x)y^{-1})x^{-1}) = yx^{-1}zxy^{-1},$$

and hence the consistency condition is satisfied. □