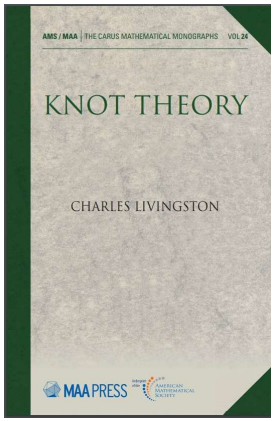
Introduction to Knot Theory

Chapter 7. Numerical Invariants

7.5. Independence of Numerical Invariants—Proofs of Theorems



Theorem 7.2

Theorem 7.2. If a knot K can be labeled with all transpositions from S_n then $\text{brg}(K) \geq n$.

Proof. Given a labeling of K with all the transpositions of S_n, we know that the set of labels generates all of S_n since every (see my online notes for Introduction to Modern Algebra [MATH 4127/5127] on II.9 Orbits, Cycles, Alternating Groups; see Corollary 9.12). The labels on the bridges (that is, the arcs with local maxima) determine all other labels, as was claimed in Chapter 5. By Exercise 5.1.8, S_n cannot be generated by fewer than $n-1$ transpositions. So the number of bridges must be at least n, as claimed. □