

Introduction to Knot Theory

Chapter 7. Numerical Invariants

7.5. Independence of Numerical Invariants—Proofs of Theorems

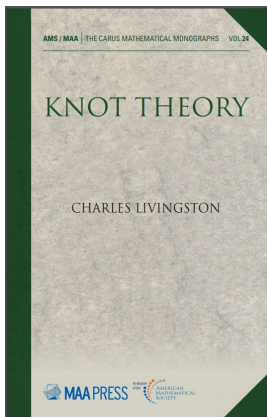


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