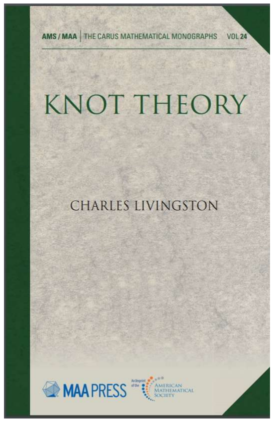
Introduction to Knot Theory

Chapter 9. High-Dimensional Knot Theory

9.5. The Knot Concordance Group—Proofs of Theorems



Lemma 9.5.6

Lemma 9.5.6. If K_1 is concordant to K_2 and J_1 is concordant to J_2, then $K_1 \# J_1$ is concordant to $K_2 \# J_2$.

Partial Proof. Livingston claims that is K and J are slice then $K \# J$ is slice; he claims this is a "geometric fact" that the "reader should be able to sketch out the argument." To show that $K_1 \# J_1$ is concordant to $K_2 \# J_2$, we need to show $(K_2 \# J_1) \# (K_2 \# J_2)^{rm}$ is slice. Now $(K_2 \# J_2)^{rm} = K_2^{rm} \# J_2^{rm}$, and $\#$ is commutative and associative (rigorous justification of these two claims should be given), so

$$(K_2 \# J_1) \# (K_2 \# J_2)^{rm} = (K_1 \# K_2^{rm}) \# (J_1 \# J_2^{rm}).$$

This is a connected sum of two slice knots (namely, the slice knots $(K_1 \# K_2^{rm})$ and $(J_1 \# J_2^{rm})$; these are slice because of the concordance hypothesis). So this knot is slice and hence $K_1 \# J_1$ and $K_2 \# J_2$ are concordant, as claimed. □

Theorem 9.5.7

Theorem 9.5.7. With respect to the operation induced by connected sum, the set of concordance classes of knots forms an abelian group, denoted C_1^3, and called the *concordance group*.

Proof. The connected sum $\#$ is associative and commutative (as claimed in the proof of Lemma 9.5.6). The identity is the concordance class of the unknot U, since $K \# U = K$ for all knots K. Now the concordance class of the unknot consists of all slice knots, because K and U are concordant if (by definition) $K \# U = K$ is slice. So the inverse of knot K is knot K^{rm} since $K \# K^{rm}$ is slice (and hence $K \# K^{rm}$ is in the equivalence class containing U). Therefore, C_1^3 is an abelian group, as claimed. □